Selection of Homework Questions

Topic 9: Gas & Dust

(1) Interstellar radiation field (ISRF)

a. Derive an expression for the radiation field, J, (erg s^{-1} cm^{-2} sr^{-1}) coming from a direction in which the luminosity density varies with distance, r, from you as $\rho_L(r)$.

b. Recover Olber's paradox by considering an infinite (static) uniform distribution of stars/galaxies.

c. What's the **intergalactic** radiation field in erg s^{-1} cm^{-2} sr^{-1} for a static Universe with radius ~ 4 Gpc (the current horizon distance) and uniform galaxy luminosity density ~$10^8 L_\odot$ Mpc^{-3} (given in Topic 4.3 < link >)

d. What's the **interstellar** radiation field near the sun (a) in the galactic plane, and (b) towards the galactic poles (i.e. our night sky brightness inside and out of the Milky Way). Assume the sun is at the mid-plane, which has a luminosity density of 0.07 $L_\odot$ pc^{-3} and exponential scale height ~100 pc. Assume the view is transparent away from the plane, but suffers 1.6 mag/kpc of attenuation in the plane.

e. A galaxy has surface brightness μ_V mag/ss. Re-express this in units of erg s^{-1} cm^{-2} sr^{-1} (the V band zero point is 3.13×10^{-6} erg $s^{-1}cm^{-2}$ at V = 0^m).

f. Since surface brightness is independent of distance, the ISRF **inside** a galaxy is roughly half the galaxy's measured surface brightness (i.e. μ + 0.75 mags/ss; there is half ahead of you, and half behind). Hence estimate the ISRF at the center of M87 (μ_V ~ 17 mag/ss) and M32 (μ_V ~ 11 mag/ss). Compare the hemispherical sky brightness in starlight in these galactic nuclei to a moonlit night on earth (take the moon's albedo to be ~2%).

(2) Equilibrium Dust Temperatures and Spectra:

In the notes, we considered the simplified case of equilibrium dust temperature. Let's return to that topic with a bit more precision.

a. Let's start simple. Write the equation for the Eddington equilibrium temperature, T_{Edd}, for a dust grain distance r from a star with temperature $T_\star$ and bolometric luminosity $L_\star$. For this approach, we assume both star and dust act like black bodies, so that $Q_{abs} = Q_{em} = 1$ at all λ. What is T_{Edd}

for dust at r = 0.1 pc from a BOV star with T_{eff} = 30,000K and $L_{bol} = 5.2 \times 10^4 L_\odot$? How does T_{Edd} depend on r ?

b. Now let's get more realistic. Consider a spherical silicate grain of radius a = 3 μm and refractive index m = 1.5 - 0.05i, which obeys the Mie relation: $Q_{abs}(\lambda) = Q_{em}(\lambda) = -4 X \, \mathrm{Im}[(m^2-1)/(m^2+2)]$ for $X < X_c$ and Q = 1 for $X > X_c$; where $X = (2\pi a/\lambda)$, and X_c is defined when Q reaches 1. What wavelength, λ_c, corresponds to X_c, beyond which the grain exhibits reduced efficiency: $Q(\lambda) \propto \lambda^{-1}$? Compare λ_c with the Wein peak wavelength for black bodies at T_{eff} and T_{Edd} (from above). How will this affect Q_{abs} and Q_{em} and hence T_d?

c. Now rewrite your original equilibrium equation, allowing for wavelength dependent $Q_{abs}(\lambda)$ and $Q_{em}(\lambda)$. Solve this equation for the above case to find the equilibrium temperature, T_d. You will need to use a numerical integrator (qromb in Numerical Recipes works fine, or use another. Don't forget to check your routine works properly!). You can either iterate numerically to find T_d or try a few values of T to find when heating = cooling. What is the "greenhouse factor", ie T_d/T_{Edd}, in this case?

d. Redo the calculation for a smaller dust grain: a = 0.03μm and explain why T_d has changed. Redo the calculation for the original (a = 3 μm) grain, but now placed at r = 3 pc from the star. Has T_d followed the r-dependence you derived in part 1 above, and if not why? Redo the calculation to find the "sublimation distance", $T_d \approx$ 1500K for the small grains.

e. For the larger grain at 0.1pc, plot its emitted spectrum (in F_λ) and compare it to a black body spectrum, B_λ, of the same temperature. If you simply took the peak wavelength to derive a dust temperature using the Wein relation, what temperature do you get? Not only are the grains hotter than the simple Eddington value, but their peak emission suggests they are even hotter than they actually are.

f. Now consider a population of silicate grains with size distribution $dN/da \propto a^{-3.5}$ between 0.03 μm (30nm) and 0.3μm (none outside this range). In this population, which grains (small or large) have most of the mass? Which have most of the area? Use numerical methods to plot the spectrum generated from the population at 0.1 pc. Use the wavelength of the peak to derive a simple single Wein temperature and overplot a black body of this temperature. Is this single temparature closer to the temperature of the larger grains or the smaller ones?

g. Finally, consider a uniform dust distrubution which extends from 1pc down to the larger of the two sublimation radii. Divide the region into 10 radial shells and sum their spectra to find the integrated IR spectrum from the region. Overplot a single black body spectrum set to the Wein temperature derived from the peak. Does all this averaging cause the spectrum to deviate significantly from a black body?

(3) Small Grain Cooling Times

a. Consider a small grain containing N=150 atoms of carbon, packed with density 1 gm cm^{-3}, which is struck by a single UV photon of energy 10eV. What's the grain's temperature immediately after the photon is absorbed?

b. Assuming the grain has refractive index m = 1.5 - 0.2i, estimate Q_{em} at λ ~ 10 μm. Estimate how long it takes the grain to cool down.

(4) Extinction & Reddening

a. Use the equations given in Cardelli , Clayton, & Mathis (1989, ApJ 345 245 e-link) to write a subroutine for A_λ/A_V and use this to generate a plots of A_λ/A_V vs $1/\lambda$ for for R_V = 2.0, 3.1, 5.0 over the range 100 nm to 3 μm. Describe, briefly, the various regions and their **physical origin**. Keep this subroutine for your future career's, it is likely you will need it sometime.

b. The spectrum of a galaxy nucleus has $H\beta$ flux of 1.2×10^{-14} erg s^{-1} cm^{-2} at observed wavelength 498nm, with other lines at relative strength to $H\beta$ of : $H\alpha$ = 4.2; [OIII]λ5007 = 12.0; [OII]λ3727 = 2.4; [OI]λ6300 = 0.20.
   - Assuming R_V = 3.1 and an unreddened Balmer ratio of $H\alpha/H\beta$ = 2.86, what is A_V?
   - Using this value of A_V, correct all the relative line strengths as well as the $H\beta$ flux.
   - Calculate the uncorrected and corrected **luminosity** of $H\beta$
   - What is the "reddening vector" for A_V = 1 in a diagram of Log([OIII]/[OII]) (x-axis) vs Log($H\alpha$/[OI]) (y-axis) (i.e. the Δx, Δy offset resulting from applying 1 magnitude of A_V to any plotted point in this diagram).

c. From B&T, find M_V, $(B-V)_o$ and $(V-K)_o$ for a B0V main sequence star. From spectra, you identify an embedded star as type, B0V, and from photometry you find B=19.67, V=16.99.
   - What is the reddening, E(B-V) ?
   - Adopting the standard extinction law, for which R_V = 3.1, what is A_V?
   - What is the unreddened apparent magnitude, V_o.
   - What is the distance modulus, m_V - M_V, and hence distance to the star?
   - If you also measure its K magnitude to be 7.25, is this consistent with the standard reddening law given in B&T, and if not, should R_V be larger or smaller? Which way does this affect your distance estimate?

(5) Dust to Gas Ratios & H columns :

(6) IR color-color diagrams :

Construct an IR color-color diagram with x/y axes $Log(S_{12}/S_{25})$ vs $Log(S_{60}/S_{100})$, where S_{12} etc are measures of f_ν at 12 μm etc (e.g. in Janskys)

a. Plot the locus of a single temperature black body, from 20K to 500K, labelling the temperatures at 100, 200,.... 500K. Obviously, you will need to evaluate the Planck function $B_\nu(T)$; make sure you evaluate B rather than B since our axes are using f evaluate B_ν rather than B_λ since our axes are using f_ν

b. Evaluate the locus of the **sum** of two black body components, one at 50K and one at 250K with **integrated** fluxes in a ratio a/b; i.e. $a/50^4 \times B_\nu(50K) + b/250^4 \times B_\nu(250K)$. Vary the relative contributions of the two, marking the locations where a/b = 0.0, 0.2, 1.0, 5.0, 100.0

c. Plot spectra for the cases a/b = 0.2 & 5.0 in the range 5 μm to 500 μm, showing the two components as well as their sum.

d. Plot the locus of power-law spectra, $f_\nu \propto \nu^\alpha$, marking values of α at -2, -1, 0, 1, 2.

e. Using NED, find the IRAS fluxes for the following galaxies and plot them on the IR color-color plot. M87 (giant elliptical); M101 (early type spiral); NGC 5548 (Seyfert 1); Markarian 3 (Seyfert 2); Arp 220 (ULIG); M82 (starburst). Comment on their differing locations.

(7) Dust heating: collisions vs radiation :

Let's revisit the claim made in the notes that dust in the ISM is heated more by radiation than particle collisions.

a. Use kinetic theory to derive, or quote, the number of particles of mean speed $<v>$ striking unit area per second. For a gas of temperature T and number density n, what is the energy flux, F_{th}, impinging on unit area? Express this in terms of the gas thermal energy density (i.e. pressure). Which component of the gas contributes most to the collisional heating and why?

b. For a radiation field, what is the relation between energy density, U_{ph} and flux F_{ph}?

c. hence, show that the ratio of collisional to radiation heating of dust in the ISM is $F_{th}/F_{ph} = 3/8 \times <v>/c \times U_{th}/U_{rad}$. Because $U_{ph} \approx U_{th}$ throughout much of the ISM, then radiation easily dominates the heating.

d. Clearly, for thermal heating to compete with radiation, we need thermal energy densities larger than radiation energy densities by a factor of roughly $c/<v>$. What is $c/<v>$ for electrons at temperature T? Are there any environments you can think of where this condition is satisfied?