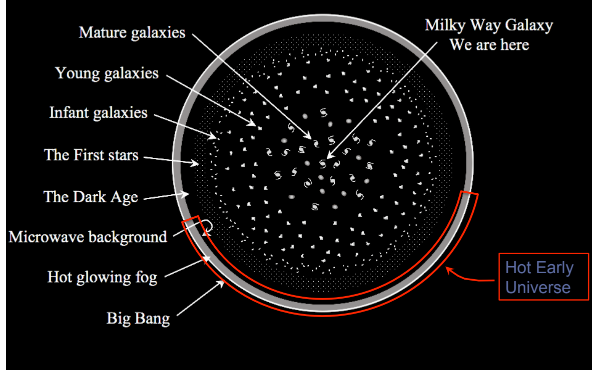
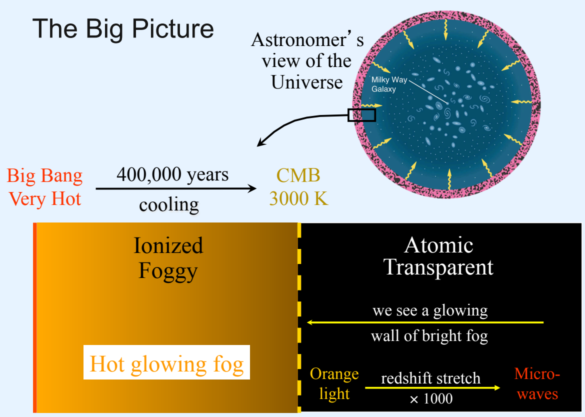
The Microwave Background

Notes based on Teaching Company lectures, and associated undergraduate text – with some additional material added.

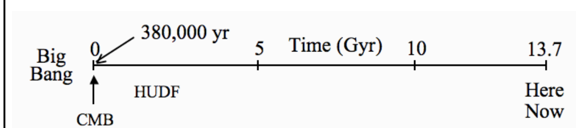
The CMB appears all around us, at great distance



The Big Picture

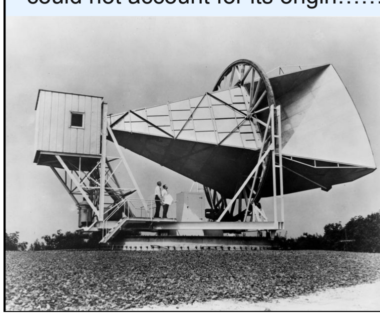


The microwave background is very "close" to the Big Bang



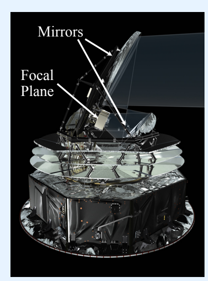
Discovery of the CMB

In 1964 Penzias and Wilson found an excess of microwave emission coming from the sky. They could not account for its origin……

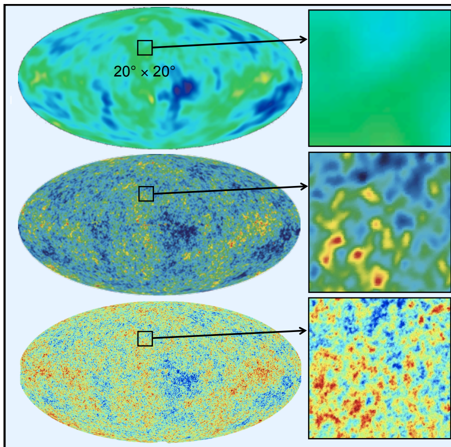


Wilson Penzias

Major CMB Space Observatories



Planck Satellite
Launched May 2009



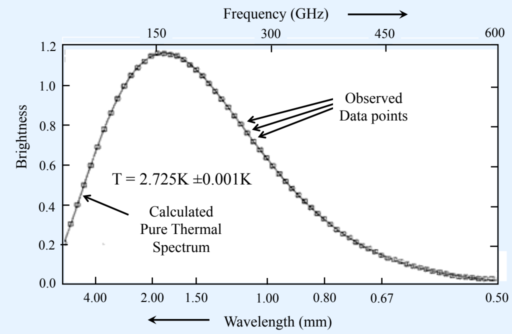
COBE: 1990
Resolution: 7°
Sensitivity: ×1
Cost: 600 M$

WMAP: 2003
Resolution: 0.13°
Sensitivity: ×5
Cost: 800 M$

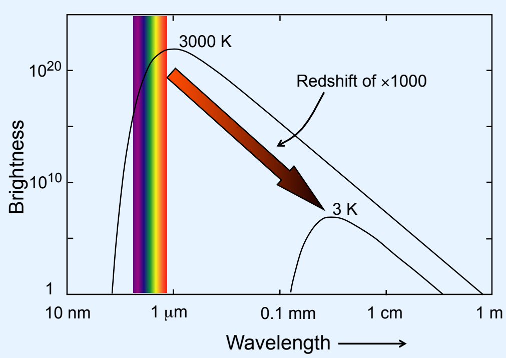
Planck: 2012
Resolution: 0.08°
Sensitivity: ×15
Cost: 1000 M$

CMB spectrum

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(h\nu/k_B T)-1}d\nu$$



Recall: redshift preserves Blackbody form with $T = T_0/a$ (see Topic 16.4g)



Temperature of recombination

Naively, recombination might happen when $k_B T \sim 13.6$ eV (typical photon energy ~ H binding energy). i.e. T ~ 150,000K.

However, photons more abundant than protons, so even at lower temperature, there are many more photons with E > 13.6 eV.

Photon/baryon ratio is large and preserved during expansion:

$$\frac{n_\gamma}{n_b} \approx \frac{\Omega_{r,0} / < E_\gamma >}{\Omega_{b,0} / < E_b >} \approx \frac{5.0\times10^{-5} / 7\times10^{-4}\ \mathrm{eV}}{0.04 / 938\ \mathrm{MeV}} \approx 1.7\times10^{9}$$

At 5700K, the tail of the black body contains as many photons with E > 13.6 eV as there are protons.

Temperature of recombination

More exactly, we use the Saha equation (for pure hydrogen):

$$\frac{1-X}{X^2} \approx 3.8\frac{n_b}{n_\gamma}\left(\frac{k_B T}{m_e c^2}\right)^{3/2}\exp\left(\frac{13.6\ \mathrm{eV}}{k_B T}\right)$$

Where X is the fractional ionization: $n_e/n_b = n_p/(n_p+n_H)$

Setting X ~ 0.5 we find T ~ 3740K, or z ~ 1370, or 240 kyr.
Transition is quick: X = 0.9 – 0.1 from z = 1475 – 1255 (Δt~70kyr)

Several complicating factors delay this time:
1) Most recombination photons are reabsorbed/reionize.
2) Reaction rates become longer than expansion rate – reactions fall out of equilibrium.

Ultimately, 2S → 1S via 2-photon decay secures recombination.
Growing m.f.p. allows Ly-α photons to redshift out of resonance.

Non-equilibrium effects delay recombination and also leave a small residual electron population ($X \sim 10^{-3}$).

This keeps the baryon temperature tied to the photon temperature until about z ~ 150 (T~ 400 K) [recall $T_{gas} \propto a^{-2}$ while $T_\gamma \propto a^{-1}$]

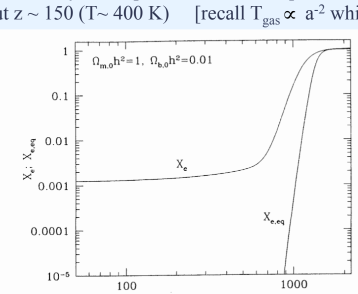


Fig. 3.8. The ionization fraction as a function of redshift, z. The curve marked $X_{e,eq}$ shows the redshift evolution of the equilibrium ionization fraction, while the one marked X_e shows the actual ionization fraction for a cosmology with $\Omega_{m,0}h^2 = 1$ and $\Omega_{b,0}h^2 = 0.01$.

Note: same physics sets ionization degree in stellar atmospheres.
G stars quite neutral; O stars: highly ionized.

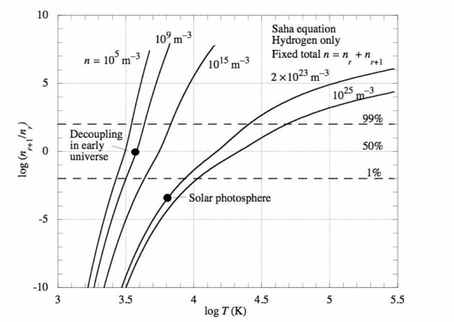


Figure 3. Plot of log ionization fraction n_{r+1}/n_r for various fixed total densities, $n = n_r + n_{r+1}$ (i.e., $n = n_H + n_p$) as a function of temperature.

Time of decoupling & last scattering

Photons Thomson scatter off free electrons, at a rate:

$$\Gamma_T = n_e \sigma_T c = X_e n_b \sigma_T c$$

When $\Gamma_T(z) > H(z)$ then the photons no longer interact (decouple). This happens around z ~ 1100 or ~3000K or 380,000 years.

This is very similar to the time of "last scattering", which is also the surface of optical depth, τ = 1, as viewed from z = 0 (here).

$$\tau(z) = \int_0^z \Gamma_T \frac{dt}{dz} dz = \int_0^z \frac{\Gamma_T(z)}{H(z)} \frac{dz}{1+z} \approx 0.37 \left(\frac{z}{1000} \right)^{14.25}$$

This also happens around z ~ 1100. There is a width Δz ~ 80 from which most of the photons last scattered.

This is the "surface" we see as the Microwave Background.

All-sky CMB image

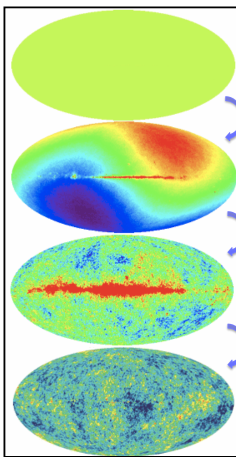
Since today's universe is lumpy on small scales, one expects to see variations in the brightness of the CMB.

After many years of searching, these were finally seen in the 1990 COBE data.

Since then, mapping these fluctuations has proved to be extremely valuable.

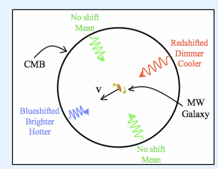
The CMB fluctuations provides our point of entry into the major topic of the origin and growth of structure.

Features visible in the CMB all-sky map



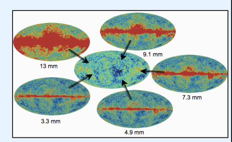
×1000 contrast

dipole

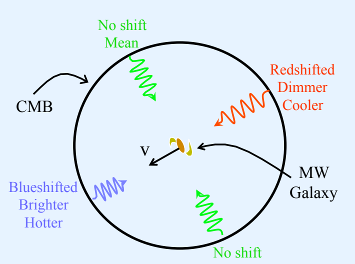


×200 contrast

remove galaxy



Doppler origin of the CMB Dipole



ΔT = 3.36 mK

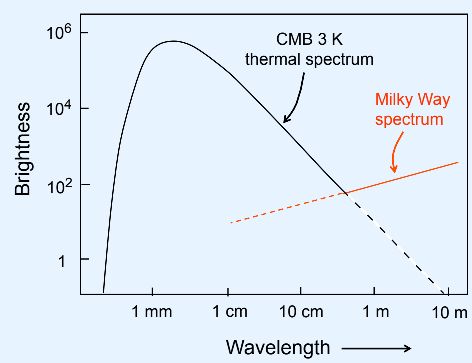
$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}$$

$V_\odot$ = 370 km/s
$\alpha = 11^h\ 12^m$ δ = –7.2°

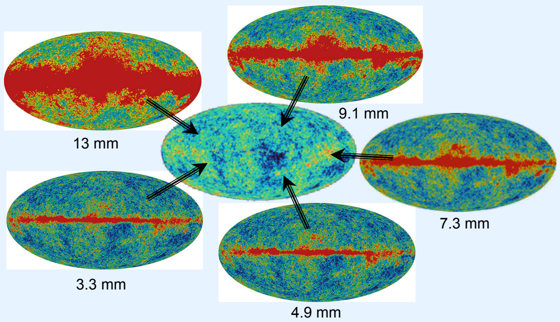
V_{LG} = 630 km/s
$\alpha = 11^h\ 06^m$ δ = –27°
Towards Hydra.

The Doppler velocity matches (approximately) an analysis of the local velocity field: ~300 km/s to Virgo + ~500km/s to Great Attractor.

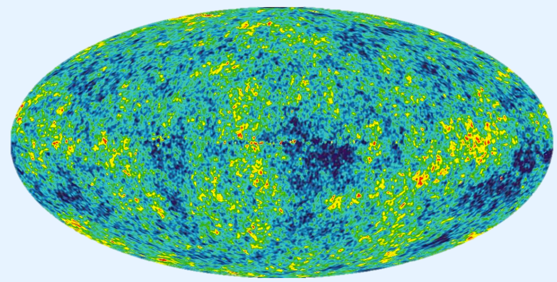
CMB and Galaxy spectra



Removing the MW foreground



Final WMAP 9-year data

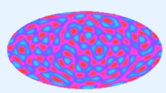


Statistical description of anisotropies

Fourer approach to functions on a sphere uses spherical harmonics:

$$\Delta T(\theta,\varphi) = \sum_{l,m} a_{l,m} Y_{l,m}(\theta,\varphi)$$

$$C(l) = \frac{1}{2l+1} \sum_{m} a^2_{l,m}$$



Example of l = 16, summed over m

Normally plot $C(l)\ l(l+1)/2\pi$ μK^2 against $l \approx 180/\Delta\theta°$.
This shows relative power per log l.

Note: this function on a surface arises (in part) from a 3-D density field, $\delta(\mathbf{r}) = \delta\rho(\mathbf{r})/\langle\rho\rangle$, which is described by $P(k)$, with $k = 2\pi/\lambda$:

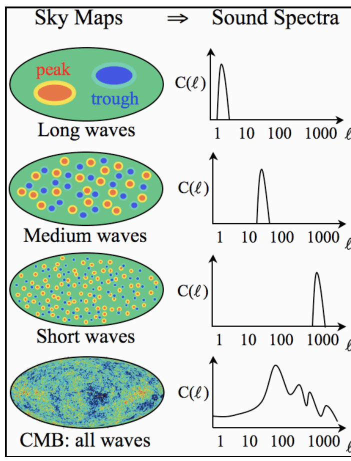
$$\delta(\vec{r}) = \sum_{k_x,k_y,k_z} \delta_k\ e^{i\vec{k}.\vec{r}}$$

Where $k = |\mathbf{k}|$ due to isotropy

$$P(k) = \langle|\delta_k|\rangle^2$$

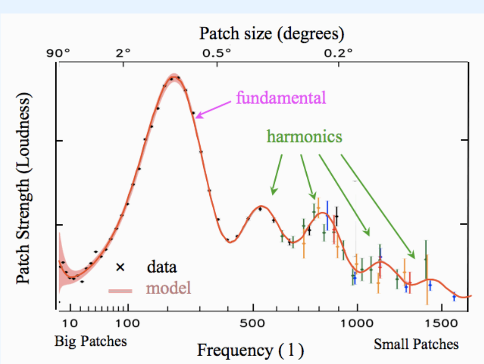
Note: primordial $P(k) \sim k^1$ has $C(l)\ l(l+1)$ = const.

Note: $P(k)$ is primordial outside horizon size & modified inside.

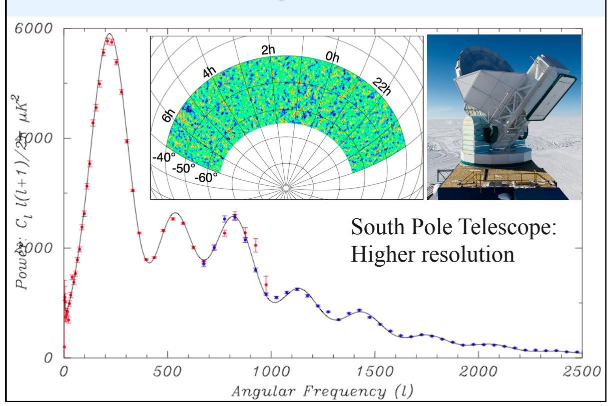


Simple depiction of how the power spectrum is constructed.

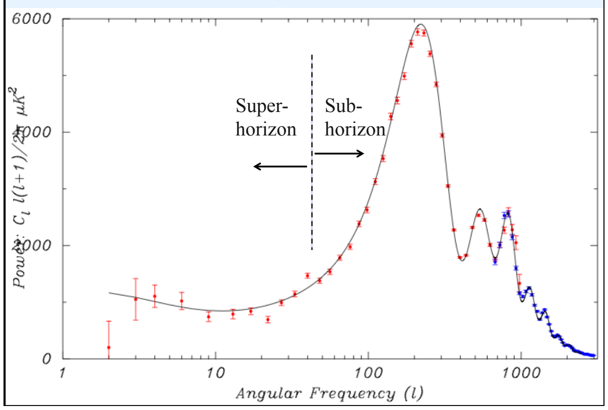
The Measured CMB Power Spectrum



WMAP-7yr + SPT-2011

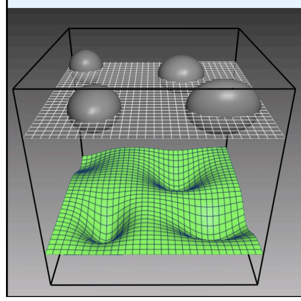


WMAP-7yr + SPT-2011



Origin of Anisotropies.

Two main constituents:
i) dark matter: almost smooth + slight density uneveness
ii) photon-baryon gas, coupled by Thomson scattering.



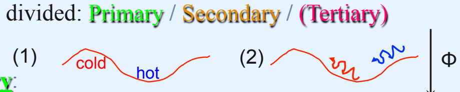
Dark matter regions create gravitational valleys.

Gas falls in and bounces out, and falls in again, etc. This is a ~spherical sound wave.

Frequency of sound wave depends on size of region.

Understanding C(ℓ)

- What are all those bumps and wiggles ??
- Many processes affect C(ℓ), not all are sound waves !! Usually divided: Primary / Secondary / (Tertiary)

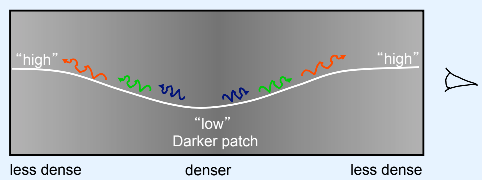


- **Primary**:
  1. more/less dense (valley/hill) → hot/cold → blue/red
  2. valley/hill → gravitational red/blue shift

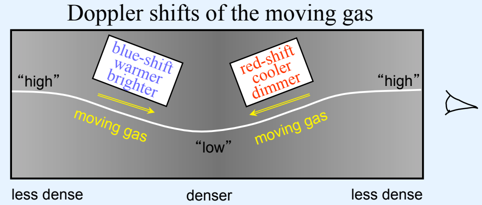
  these partly cancel: $\frac{\Delta T}{T} = \frac{\Phi}{c^2}$ & $\frac{1}{3}\frac{\Delta\rho}{\rho}$ ⇒ $\frac{\Delta T}{T} = \frac{2}{3}\frac{\Phi}{c^2}$ Sachs Wolfe
  (sub & super horizon)
  3. gas falls in / rebounds out → Doppler red/blue shift
  (sub-horizon only, ie ℓ > 50)

Formation of fundamental & harmonics →

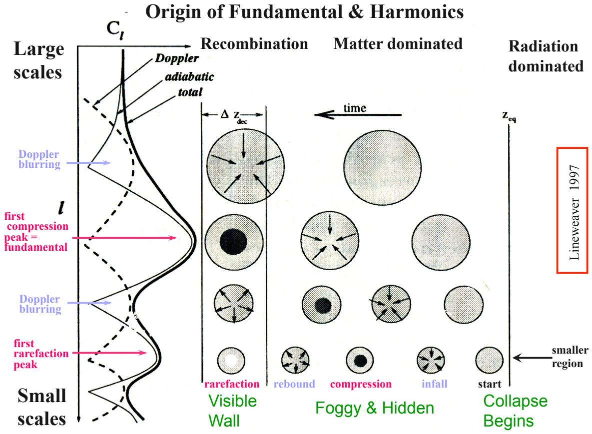
Gravitational Redshift: Sachs-Wolfe Effect

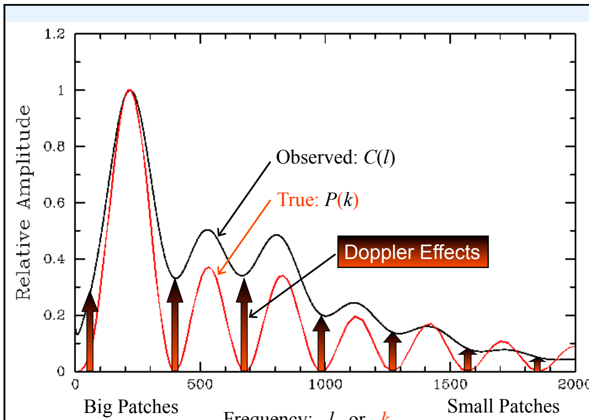


Doppler shifts of the moving gas



Origin of Fundamental & Harmonics

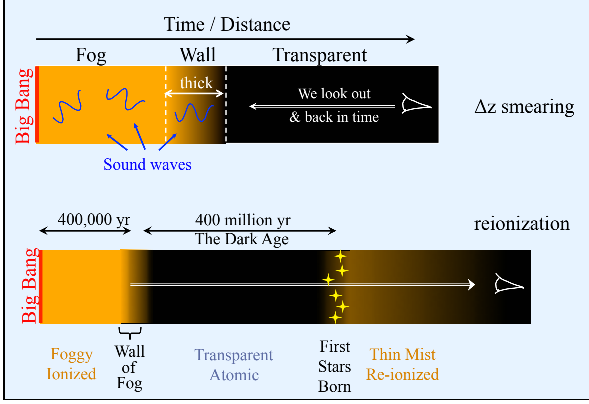


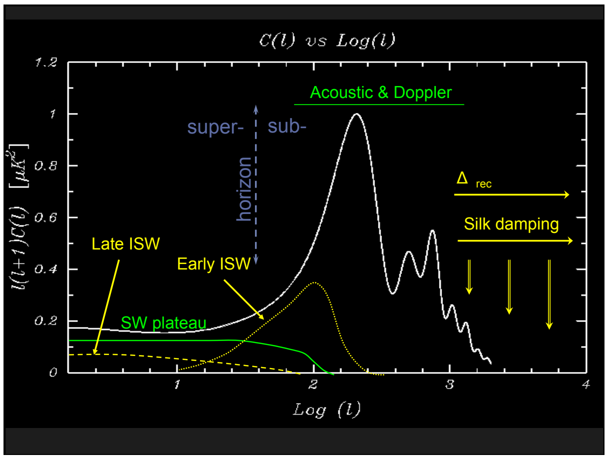


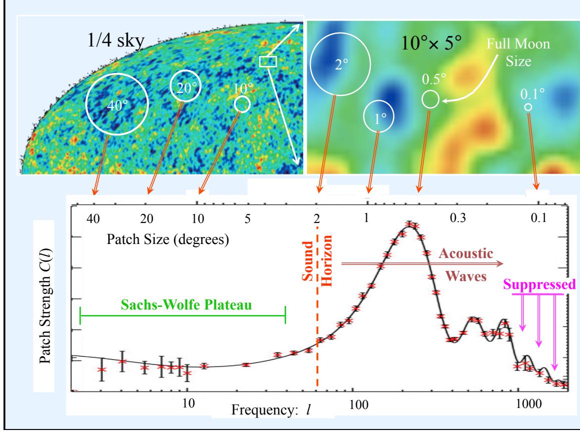
Understanding C(ℓ)

- **Secondary**:
  1. Smearing : $\lambda < \Delta R_{rec}$ wash out; kills high ℓ
  2. Silk damping: photon diffusion, kills high ℓ
  3. Integrated Sachs-Wolfe: γs cross varying Φ
     Early ISW : $\dot{\Phi}_\gamma$ near z_{rec} (adds power @ ℓ~100)
     Late ISW : $\Lambda \to \Phi(z<1)$ (adds power @ ℓ<10)
  4. Re-ionization : @ z~20 lowers power for $\lambda < \lambda_H(z_{re\text{-}ion})$
  5. Cluster SZ effects : add power @ high ℓ >3000
- **Tertiary** (contamination):
  1. Galactic: dust, free-free, synchrotron
  2. Point sources: radio galaxies; high-z IR gals
  3. Dipole ($10^{-3} \times T_{cmb}$; $10^2 \times$ other anisotropies)

Two influences at high ℓ







Diagnostics: Measuring Cosmic Parameters

Several key datasets are sensitive to the cosmological parameters. These include:

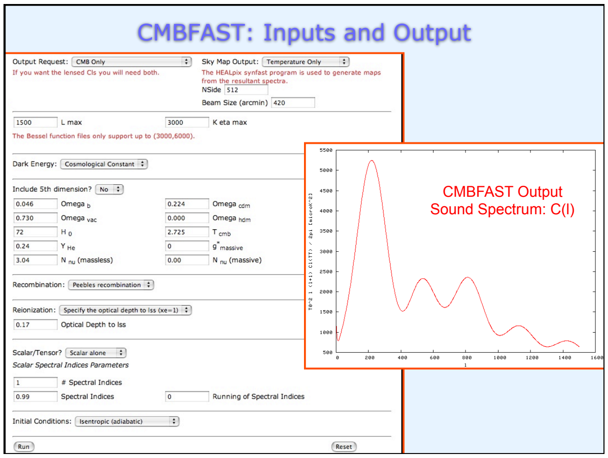
1) The CMB temperature, and power spectrum C(*l*)
2) The near-field redshift-distance data, giving H_0.
3) The far-field SNIa redshift-magnitude data.
4) The galaxy power spectrum, including BAOs.
5) The galaxy cluster number vs redshift relation.
6) The abundances of the light elements.

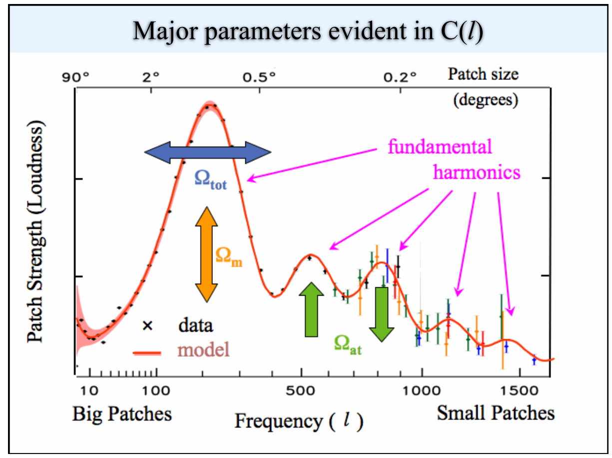
While each dataset primarily measures one or two parameters, they are normally combined to introduce redundancy, and reduce uncertainties in the parameter values.

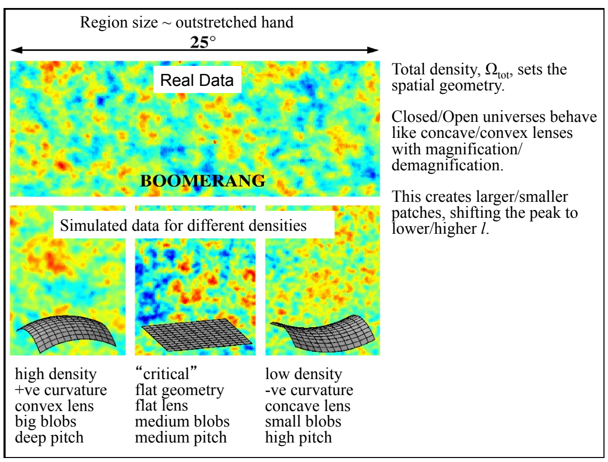
For some of these – e.g. C(*l*), P(*k*) – one needs a detailed working model to compare to data.

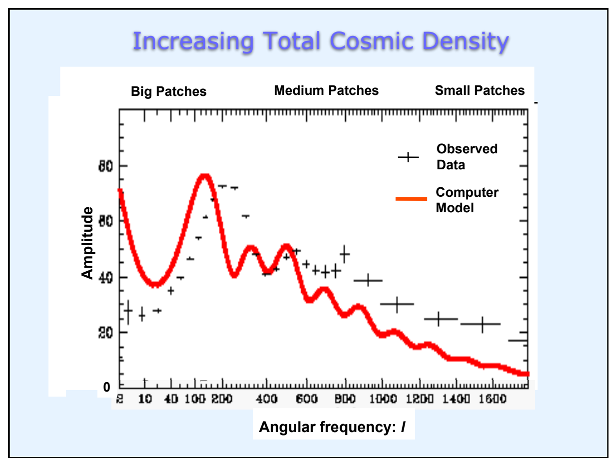
Modeling C(ℓ) and P(k)

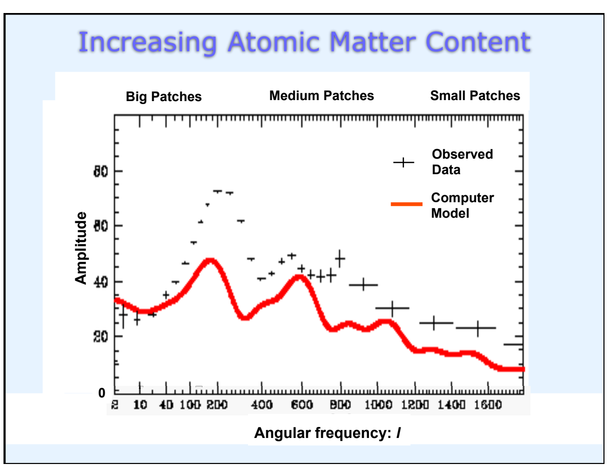
- Highly sophisticated; long history:
  Early work: Peebles; Silk; Bond; Efstathiou…..
  Developments: Seljak; Sugiyama; Zaldarriaga; Hu……..
  → Improved numerical methods & physical understanding
  → Public code : CMBFAST (Seljak & Zaldarriaga '96)
- In theory:
  4 fluids: CDM & υs (collisionless); baryons & γs (collisional)
  Evolve fluid DFs using Boltzmann Eqn + perturbations
  → P(k,z) & Transfer functions → C(ℓ)
  → Physics "known" → accurate to ≤ few %
- In practice:
  Input global: $\Omega_b\ \Omega_m\ \Omega_\Lambda\ \Omega_\nu\ T_{cmb}$ h; & perturbations: n, A, type.
  Let rip → P(k,z) for baryons, γ, cdm, υ; + C(ℓ)

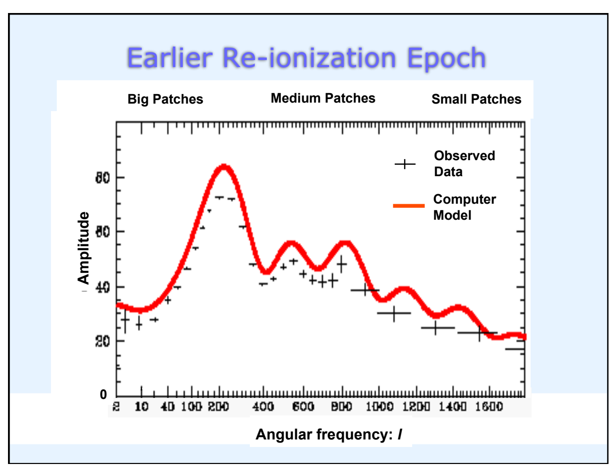

CMBFAST: Inputs and Output
CMBFAST Output
Sound Spectrum: C(l)

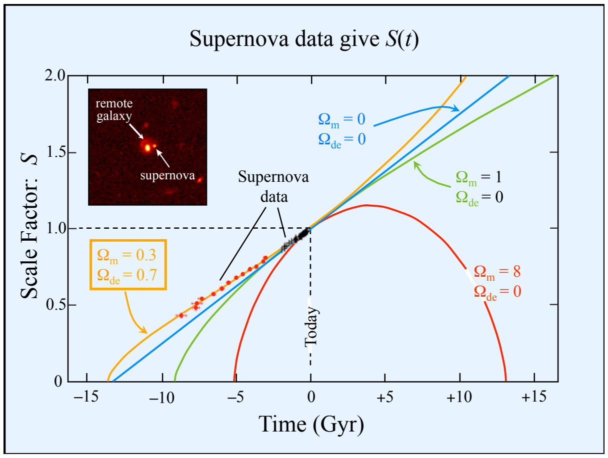

Major parameters evident in C(l)
90°
2°
0.5°
0.2°
Patch size
(degrees)
fundamental
harmonics
Ωtot
Ωm
Ωat
× data
model
Patch Strength (Loudness)
10
100
500
1000
1500
Big Patches
Frequency (l)
Small Patches

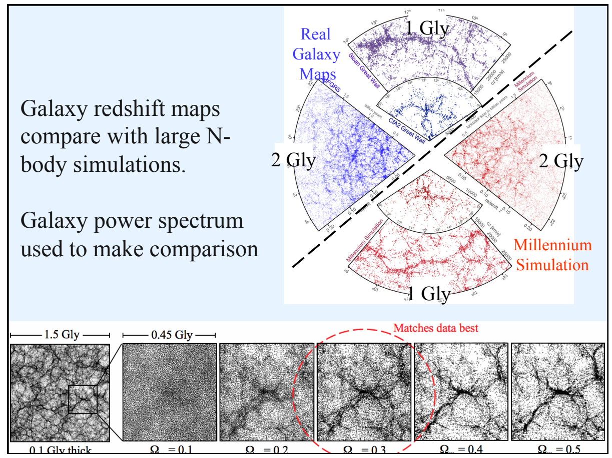

Region size ~ outstretched hand
25°
Real Data
BOOMERANG
Simulated data for different densities
Total density, Ωtot, sets the spatial geometry.
Closed/Open universes behave like concave/convex lenses with magnification/ demagnification.
This creates larger/smaller patches, shifting the peak to lower/higher l.
high density
+ve curvature
convex lens
big blobs
deep pitch
"critical"
flat geometry
flat lens
medium blobs
medium pitch
low density
-ve curvature
concave lens
small blobs
high pitch

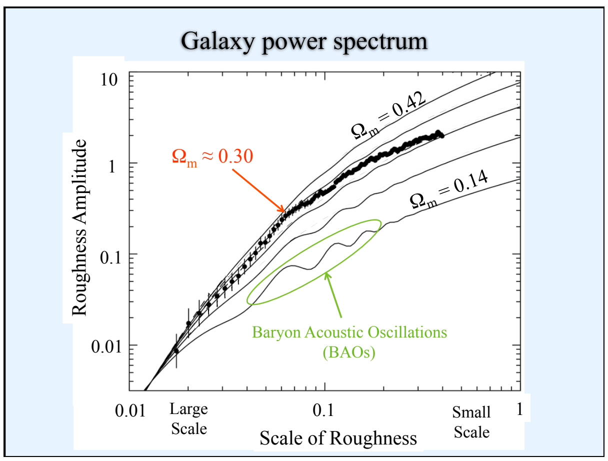

Increasing Total Cosmic Density
Big Patches
Medium Patches
Small Patches
Observed Data
Computer Model
Amplitude
Angular frequency: l

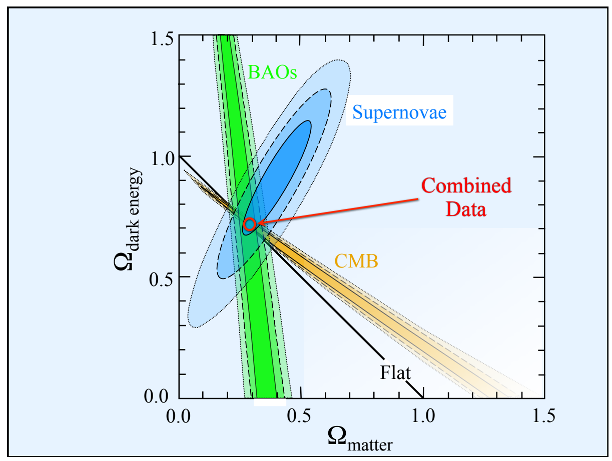

Increasing Atomic Matter Content
Big Patches
Medium Patches
Small Patches
Observed Data
Computer Model
Amplitude
Angular frequency: l

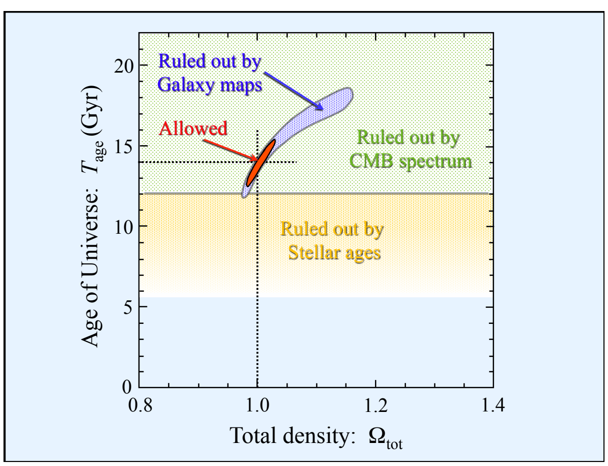

Earlier Re-ionization Epoch
Big Patches
Medium Patches
Small Patches
Observed Data
Computer Model
Amplitude
Angular frequency: l











Representative parameter values

Best-fit cosmological parameters from WMAP nine-year results[8]

Parameter	Symbol	Best fit (WMAP only)	Best fit (WMAP + eCMB + BAO + H_0)
Age of the universe (Ga)	t_0	13.74 ±0.11	13.772 ±0.059
Hubble's constant (km/Mpc·s)	H_0	70.0 ±2.2	69.32 ±0.80
Baryon density	Ω_b	0.0463 ±0.0024	0.046 28 ±0.000 93
Physical baryon density	$\Omega_b h^2$	0.022 64 ±0.000 50	0.022 23 ±0.000 33
Cold Dark matter density	Ω_c	0.233 ±0.023	$0.2402^{+0.0088}_{-0.0087}$
Physical cold dark matter density	$\Omega_c h^2$	0.1138 ±0.0045	0.1153 ±0.0019
Dark energy density	Ω_Λ	0.721 ±0.025	$0.7135^{+0.0095}_{-0.0096}$
Density fluctuations at $8h^{-1}$ Mpc	σ_8	0.821 ±0.023	$0.820^{+0.013}_{-0.014}$
Scalar spectral index	n_s	0.972 ±0.013	0.9608 ±0.0080
Reionization optical depth	τ	0.089 ±0.014	0.081 ±0.012
Curvature	$1 - \Omega_{tot}$	$-0.037^{+0.044}_{-0.042}$	$-0.0027^{+0.0039}_{-0.0038}$
Tensor-to-scalar ratio ($k_0 = 0.002$ Mpc^{-1})	r	< 0.38 (95% CL)	< 0.13 (95% CL)
Running scalar spectral index	$dn_s/dlnk$	-0.019 ±0.025	-0.023 ±0.011