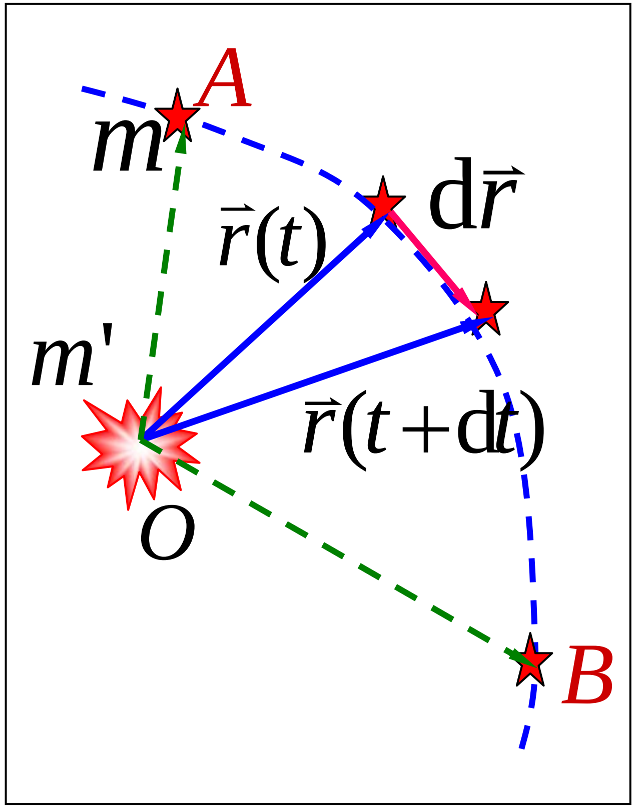
一　万有引力、重力、弹性力作功的特点

1）　万有引力作功

以m'为参考系，m的位置矢量为$\vec{r}$.

m' 对 m 的万有引力为

$$\vec{F}=-G\frac{m'm}{r^3}\vec{r}$$



m由A点移动到B点时 $\vec{F}$ 作功为

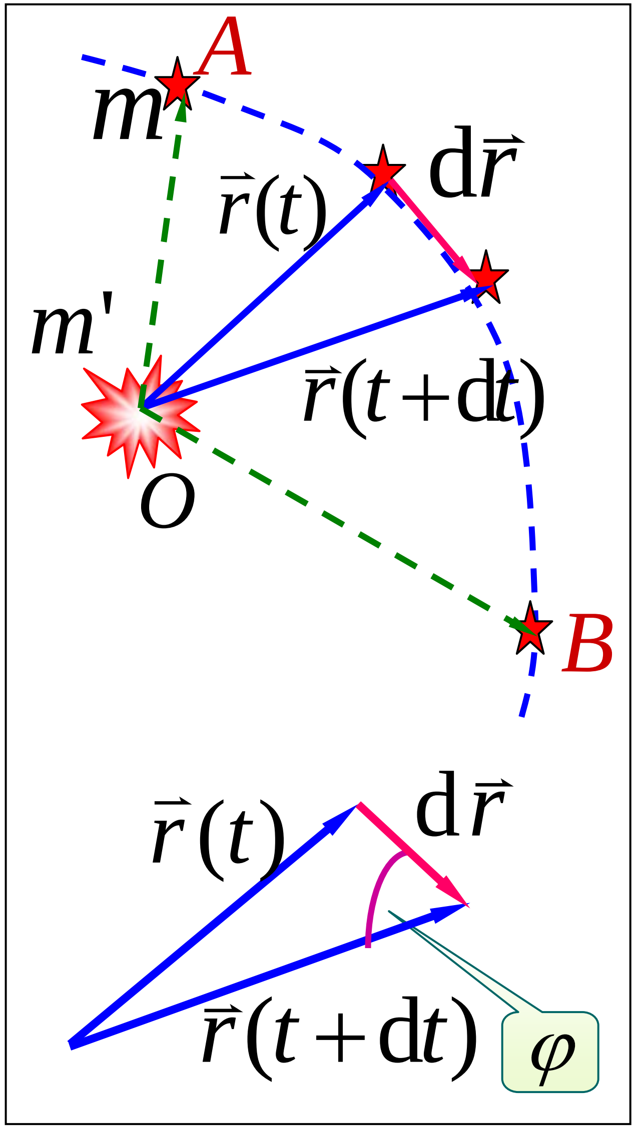
$$W=\int\vec{F}\cdot\mathrm{d}\vec{r}=\int_A^B-G\frac{m'm}{r^3}\vec{r}\cdot\mathrm{d}\vec{r}$$

$$W=\int \vec{F}\cdot \mathrm{d}\vec{r}=\int_{A}^{B}-G\frac{m'm}{r^{3}}\vec{r}\cdot \mathrm{d}\vec{r}$$

$$\vec{r}\cdot \mathrm{d}\vec{r}=r\left|\mathrm{d}\vec{r}\right|\cos\varphi=r\mathrm{d}r$$

$$W=\int_{r_A}^{r_B}-G\frac{m'm}{r^{2}}\mathrm{d}r$$

$$W=-\left[(-G\frac{m'm}{r_B})-(-G\frac{m'm}{r_A})\right]$$



2） 重力作功

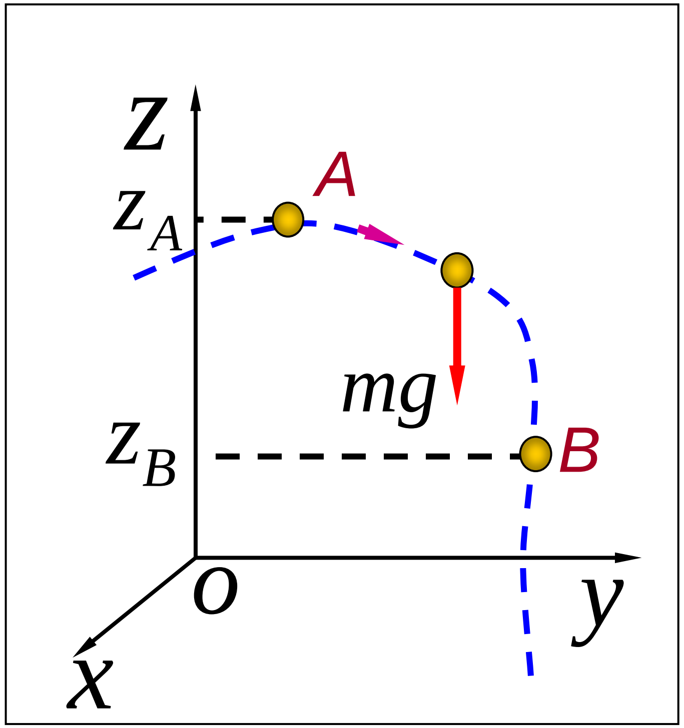
$$\boxed{\vec{P} = -mg\vec{k}}$$

$$\mathrm{d}\vec{r} = \mathrm{d}x\vec{i} + \mathrm{d}y\vec{j} + \mathrm{d}z\vec{k}$$

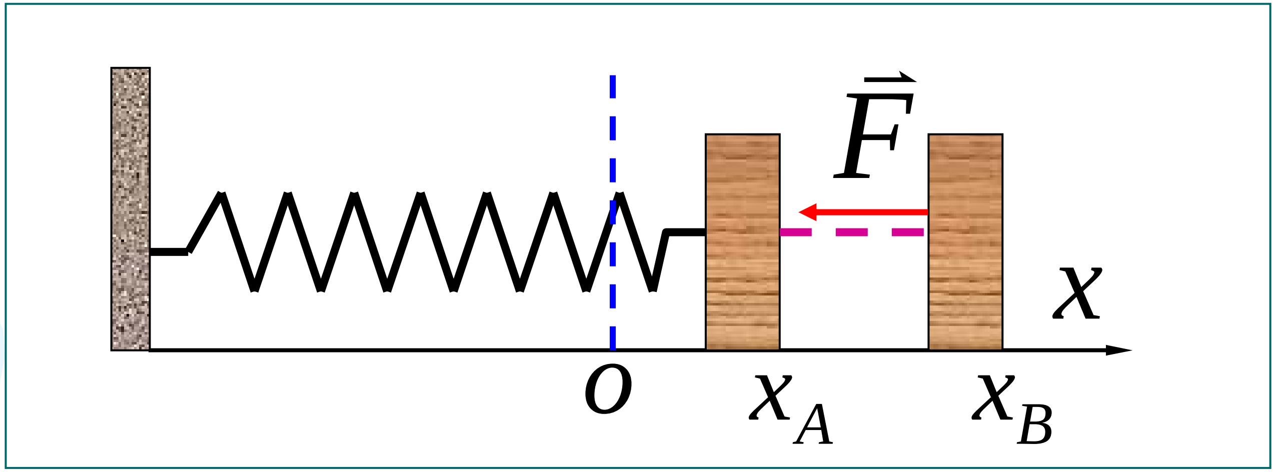
$$W = \int_A^B \vec{P} \cdot \mathrm{d}\vec{r} = \int_{z_A}^{z_B} -mg\,\mathrm{d}z$$

$$= -(mgz_B - mgz_A)$$

$$W = \oint -mg\,\mathrm{d}z = 0$$



3）　弹性力作功



$$\boxed{\vec{F} = -kx\vec{i}}$$

$$W = \int_{x_A}^{x_B} F\mathrm{d}x = \int_{x_A}^{x_B} -kx\mathrm{d}x$$

$$W = -(\frac{1}{2}kx_B^2 - \frac{1}{2}kx_A^2)$$

$$W = \oint -kx\mathrm{d}x = 0$$

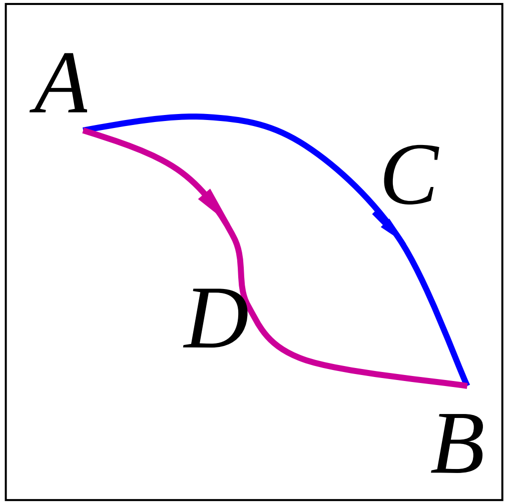
二　保守力和非保守力

保守力：　力所作的功与路径无关，仅决定于相互作用质点的始末相对位置 .

引力功　$W=-\left[(-G\dfrac{m'm}{r_B})-(-G\dfrac{m'm}{r_A})\right]$

重力功　$W=-(mgz_B-mgz_A)$

弹力功　$W=-(\dfrac{1}{2}kx_B^2-\dfrac{1}{2}kx_A^2)$

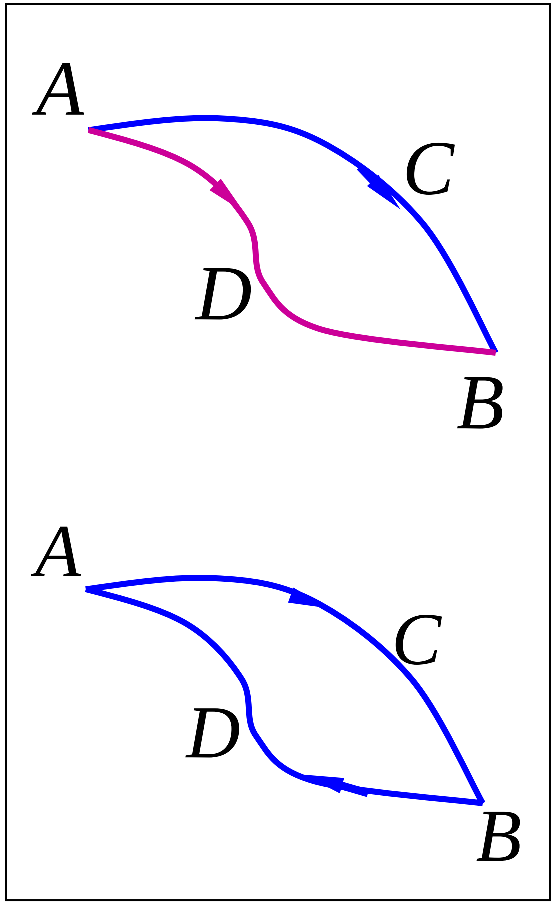


$$\int_{ACB}\vec{F}\cdot\mathrm{d}\vec{r}=\int_{ADB}\vec{F}\cdot\mathrm{d}\vec{r}$$

$$\int_{ACB}\vec{F}\cdot\mathrm{d}\vec{r}=\int_{ADB}\vec{F}\cdot\mathrm{d}\vec{r}$$

$$\oint_{l}\vec{F}\cdot\mathrm{d}\vec{r}=\int_{ACB}\cancel{\vec{F}\cdot\mathrm{d}\vec{r}}+\int_{BDA}\cancel{\vec{F}\cdot\mathrm{d}\vec{r}}$$

$$\oint_{l}\vec{F}\cdot\mathrm{d}\vec{r}=0$$



物体沿闭合路径运动 一周时，保守力对它所作的功等于零．

非保守力：力所作的功与路径有关．（例如摩擦力）

三　势能

- 势能　与物体间相互作用及相对位置有关的能量．

重力功

$$W = -(mgz_B - mgz_A)$$

引力功

$$W = -\left[\left(-G\frac{m'm}{r_B}\right) - \left(-G\frac{m'm}{r_A}\right)\right]$$

弹力功

$$W = -\left(\frac{1}{2}kx_B^2 - \frac{1}{2}kx_A^2\right)$$

重力势能

$$E_{\mathrm{p}} = mgz$$

引力势能

$$E_{\mathrm{p}} = -G\frac{m'm}{r}$$

弹性势能

$$E_{\mathrm{p}} = \frac{1}{2}kx^2$$

- 保守力的功　$W = -(E_{\mathrm{p2}} - E_{\mathrm{p1}}) = -\Delta E_{\mathrm{p}}$

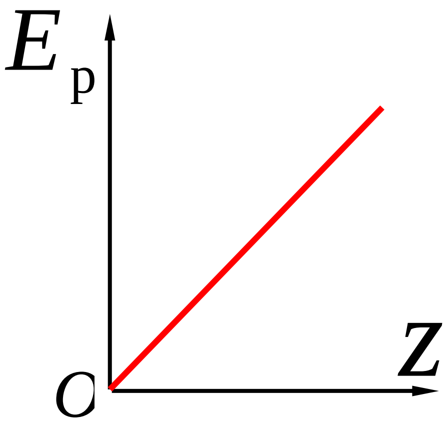
讨论

- 势能是状态函数　　$E_{\mathrm{p}} = E_{\mathrm{p}}(x, y, z)$
- 势能具有相对性，势能大小与势能零点的选取有关．
- 势能是属于系统的．
- 势能计算　　$\boxed{W = -(E_{\mathrm{p}} - E_{\mathrm{p0}}) = -\Delta E_{\mathrm{p}}}$

令 $E_{\mathrm{p0}} = 0$　　$\boxed{E_{\mathrm{p}}(x, y, z) = \int_{(x,y,z)}^{E_{\mathrm{p0}}=0} \vec{F} \cdot \mathrm{d}\vec{r}}$

四 势能曲线

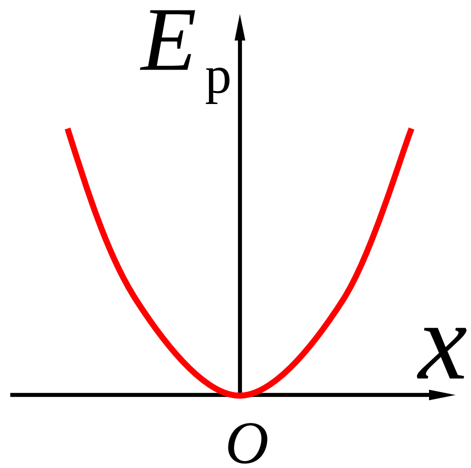
$$E_{\mathrm{p}} = mgz$$



重力势能曲线

$z = 0, \quad E_{\mathrm{p}} = 0$

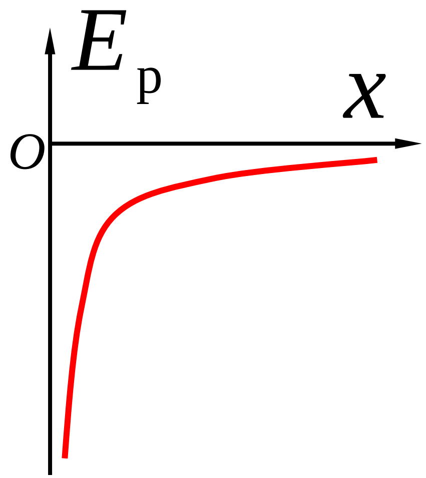
$$E_{\mathrm{p}} = \frac{1}{2}kx^{2}$$



弹性势能曲线

$x = 0, \quad E_{\mathrm{p}} = 0$

$$E_{\mathrm{p}} = -G\frac{m'm}{r}$$



引力势能曲线

$r \to \infty, \quad E_{\mathrm{p}} = 0$