一　等温过程

特征　$T=$ 常量

过程方程　$pV=$ 常量

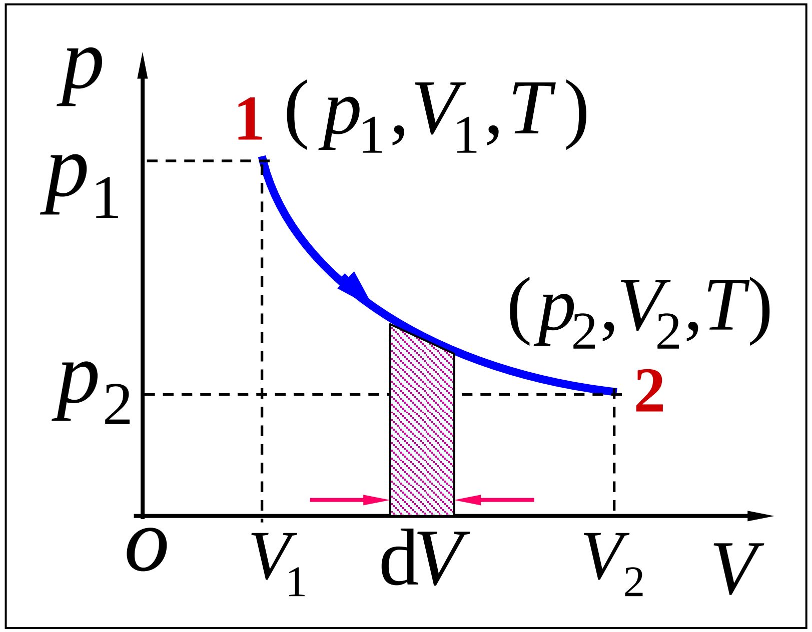
$$\mathrm{d}E=0$$

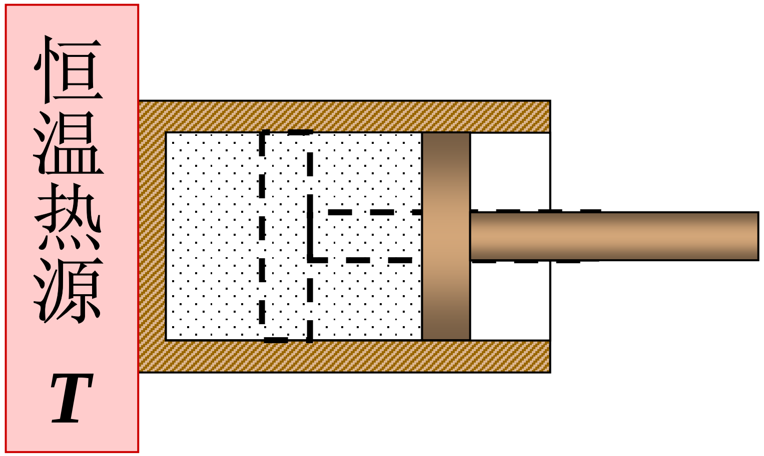
热力学第一定律

$$\mathrm{d}Q_T=\mathrm{d}W=p\mathrm{d}V$$

$$Q_T=W=\int_{V_1}^{V_2}p\mathrm{d}V$$

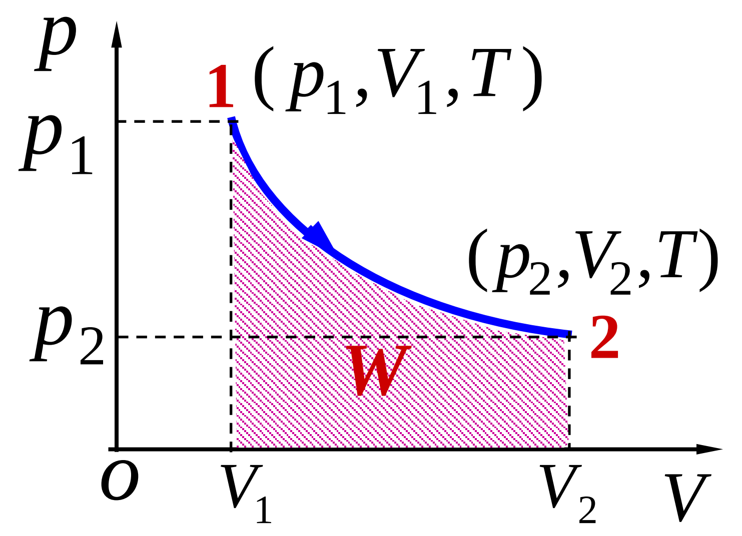
$$p=\frac{m}{M}\frac{RT}{V}$$



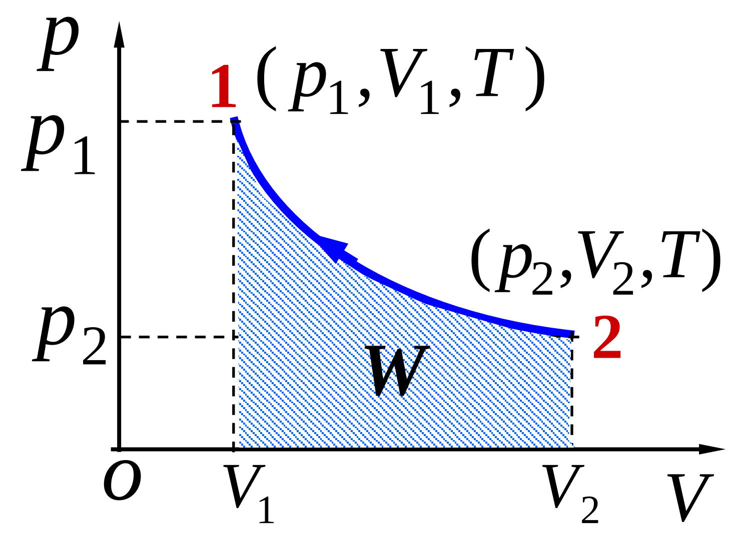


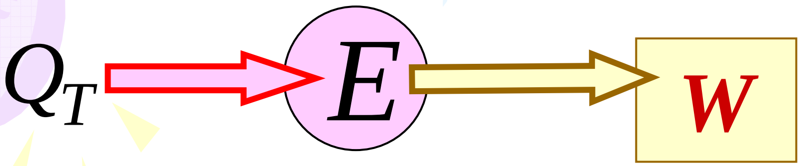
$$Q_T = W = \int_{V_1}^{V_2} \frac{m}{M}\frac{RT}{V}\,\mathrm{d}V = \boxed{\frac{m}{M}RT\ln\frac{V_2}{V_1}} = \frac{m}{M}RT\ln\frac{p_1}{p_2}$$

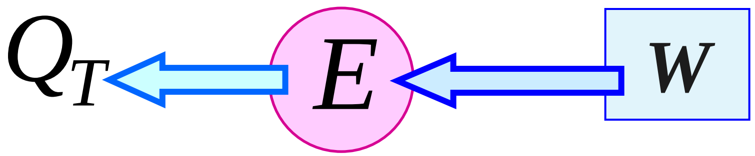
等温膨胀



等温压缩







二　绝热过程

与外界无热量交换的过程

特征　　$\mathrm{d}Q = 0$

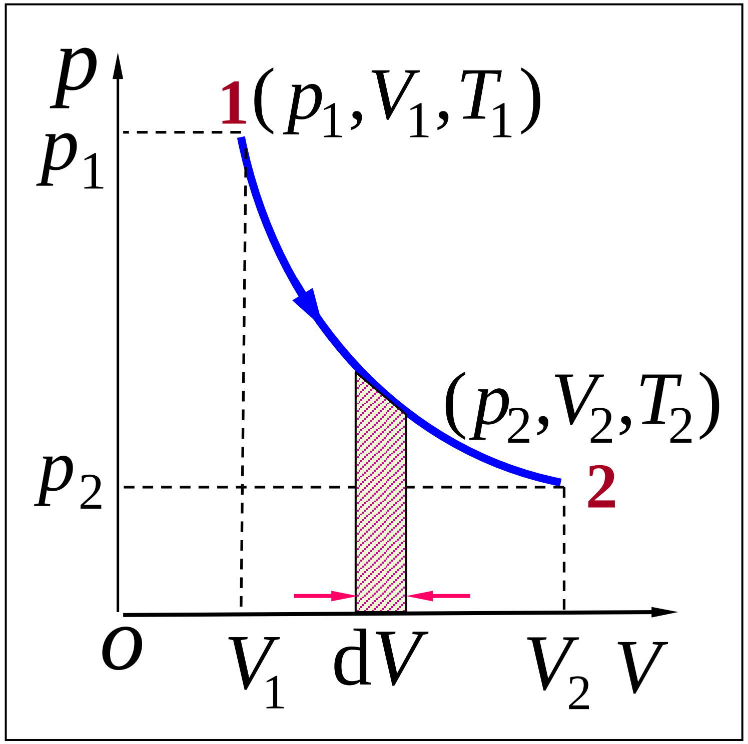
热一律　　$\mathrm{d}W + \mathrm{d}E = 0$

$$\mathrm{d}W = -\mathrm{d}E$$

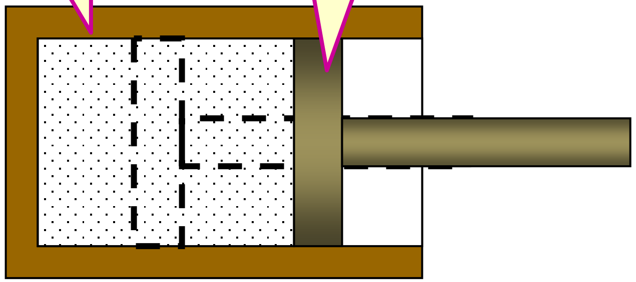
$$\mathrm{d}E = \frac{m}{M} C_{V,\mathrm{m}} \mathrm{d}T$$

$$W = \int_{V_1}^{V_2} p\mathrm{d}V = -\int_{T_1}^{T_2} \frac{m}{M} C_{V,\mathrm{m}} \mathrm{d}T$$

$$= -\frac{m}{M} C_{V,\mathrm{m}} (T_2 - T_1)$$



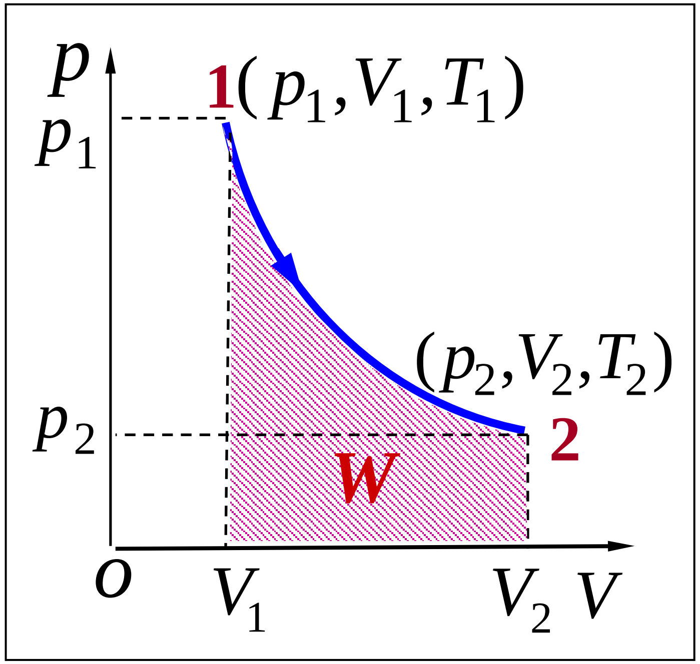
绝热的汽缸壁和活塞



由热力学第一定律有

$$W = -\Delta E$$

$$W = \frac{m}{M} C_{V,\mathrm{m}} (T_1 - T_2)$$



若已知 p_1, V_1, p_2, V_2 及 γ

从 $pV = \frac{m}{M} RT$ 可得 $W = C_{V,\mathrm{m}} \left(\frac{p_1 V_1}{R} - \frac{p_2 V_2}{R} \right)$

$$W = \frac{C_{V,\mathrm{m}}}{C_{P,\mathrm{m}} - C_{V,\mathrm{m}}} (p_1 V_1 - p_2 V_2)$$

$$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

◆ 绝热过程方程的推导

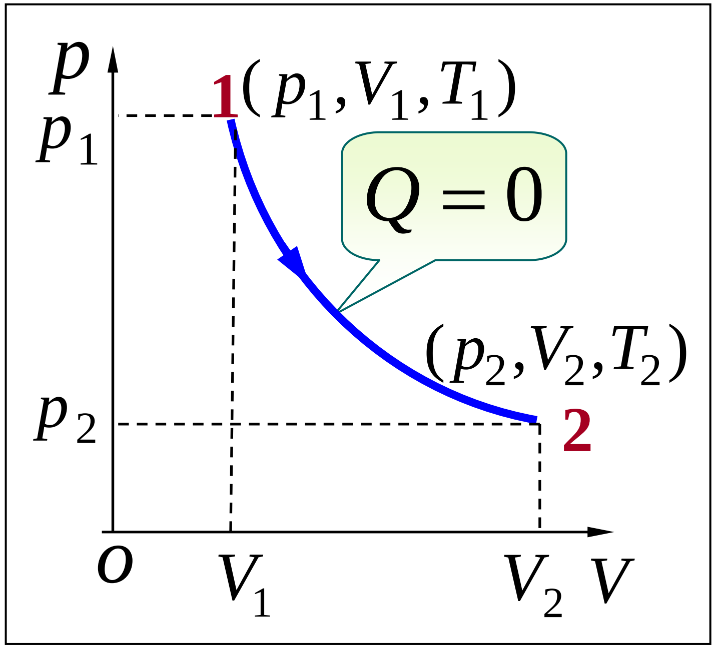
$$\because \mathrm{d}Q = 0, \quad \therefore \mathrm{d}W = -\mathrm{d}E$$

$$\begin{cases} p\,\mathrm{d}V = -\dfrac{m}{M} C_{V,\mathrm{m}} \mathrm{d}T \\ pV = \dfrac{m}{M} RT \end{cases}$$

$$\frac{m}{M}\frac{RT}{V}\mathrm{d}V = -\frac{m}{M} C_{V,\mathrm{m}} \mathrm{d}T$$

分离变量得 $\dfrac{\mathrm{d}V}{V} = -\dfrac{C_{V,\mathrm{m}}}{R}\dfrac{\mathrm{d}T}{T}$

$$\int \frac{\mathrm{d}V}{V} = -\int \frac{1}{\gamma - 1}\frac{\mathrm{d}T}{T}$$

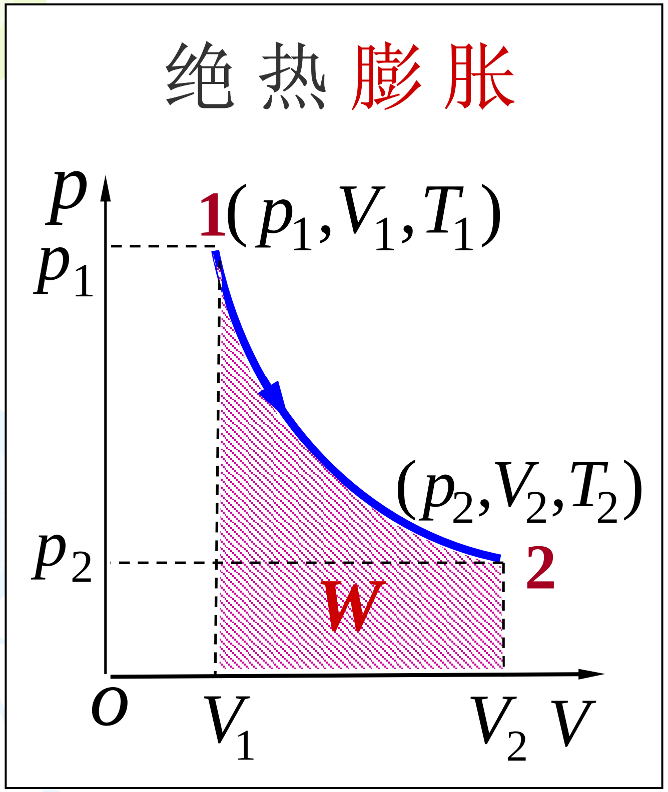


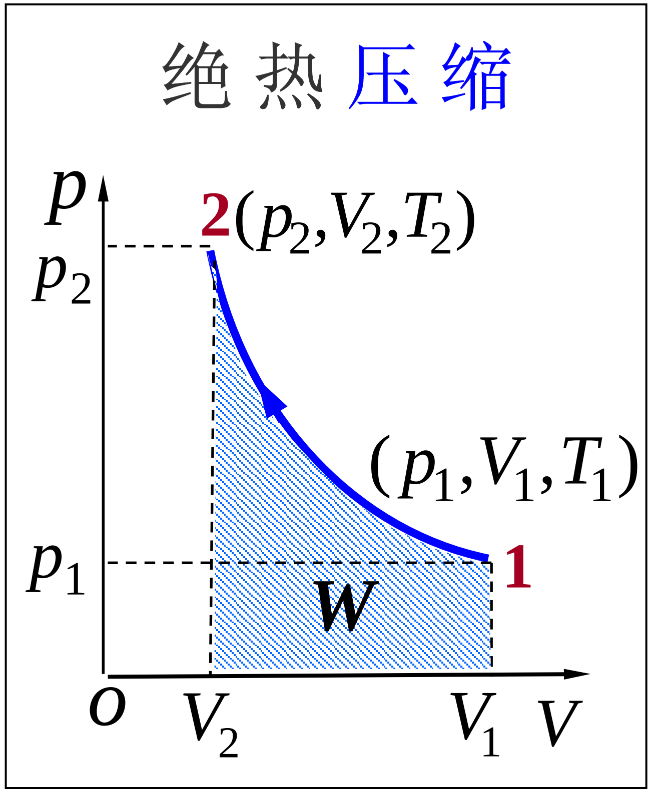
绝热方程

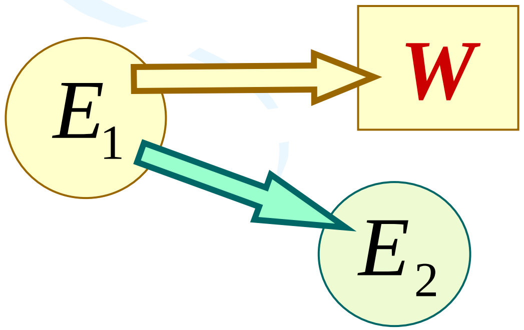
$$V^{\gamma-1}T = 常量$$

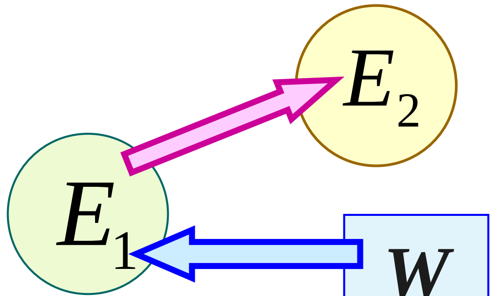
$$pV^{\gamma} = 常量$$

$$p^{\gamma-1}T^{-\gamma} = 常量$$

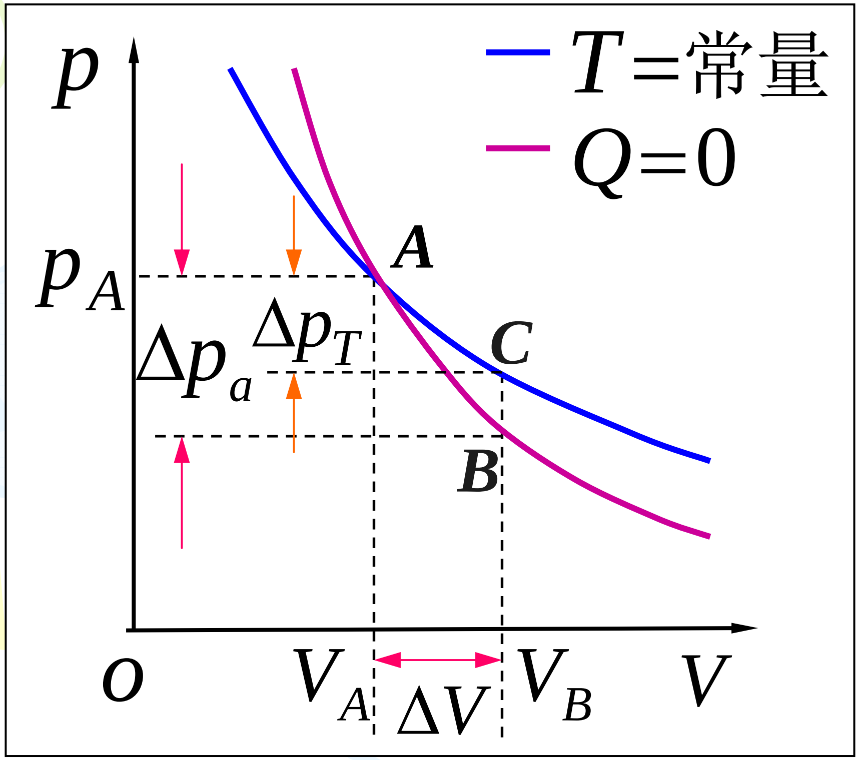

绝 热 膨 胀
p
1(p1,V1,T1)
p1
(p2,V2,T2)
p2
2
W
O
V1
V2
V


绝 热 压 缩
p
2(p2,V2,T2)
p2
(p1,V1,T1)
p1
1
W
O
V2
V1
V


E1
W
E2


E2
E1
W

三　绝热线和等温线



绝热线的斜率大于等温线的斜率.

绝热过程曲线的斜率

$$pV^{\gamma}=\text{常量}$$

$$\gamma pV^{\gamma-1}\mathrm{d}V+V^{\gamma}\mathrm{d}p=0$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}V}\right)_a=-\gamma\frac{p_A}{V_A}$$

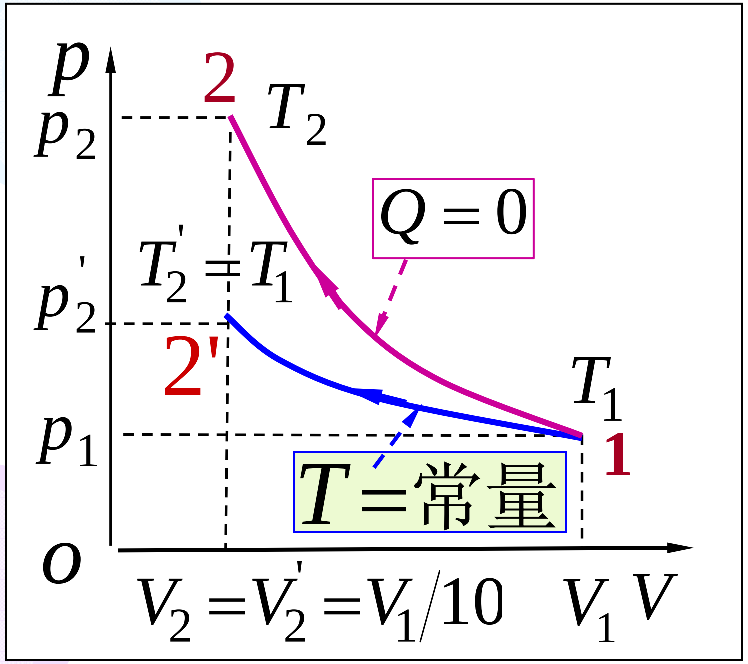
等温过程曲线的斜率

$$pV=\text{常量}$$

$$p\mathrm{d}V+V\mathrm{d}p=0$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}V}\right)_T=-\frac{p_A}{V_A}$$

例1 设有 5 mol 的氢气，最初的压强为 1.013×10^5 Pa，温度为 20°，求在下列过程中，把氢气压缩为原体积的 1/10 需作的功：**1**）等温过程，**2**）绝热过程．**3**）经这两过程后，气体的压强各为多少？



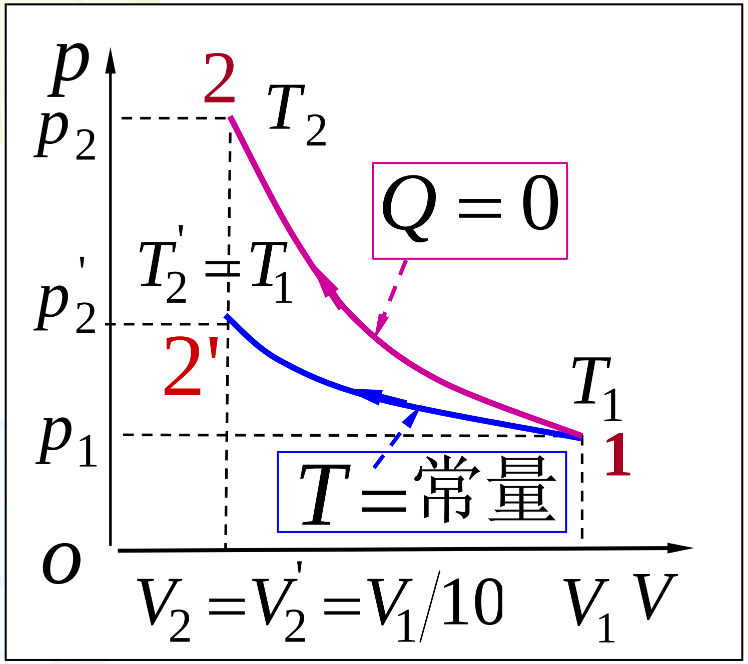
解 **1**）等温过程

$$W_{12}'=\frac{m}{M}RT\ln\frac{V_2'}{V_1}=-2.80\times10^4\,\mathrm{J}$$

2）氢气为双原子气体

由表查得 $\gamma=1.41$，有

$$T_2=T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}=753\,\mathrm{K}$$



$$T_2 = 753\text{K}$$

$$W_{12} = -\frac{m}{M} C_{V,m}(T_2 - T_1)$$

$$C_{V,m} = 20.44\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$$

$$W_{12} = -4.70 \times 10^4\,\text{J}$$

3）对等温过程

$$p'_2 = p_1\left(\frac{V_1}{V_2}\right) = 1.013 \times 10^6\,\text{Pa}$$

对绝热过程, 有 $$p_2 = p_1\left(\frac{V_1}{V_2}\right)^\gamma = 2.55 \times 10^6\,\text{Pa}$$

例2　氮气液化，　把氮气放在一个绝热的汽缸中. 开始时,氮气的压强为50个标准大气压、温度为300K;经急速膨胀后,其压强降至 1个标准大气压,从而使氮气液化 . 试问此时氮的温度为多少 ?

解　氮气可视为理想气体, 其液化过程为绝热过程.

$$p_1 = 50 \times 1.013 \times 10^5 \, \mathrm{Pa} \qquad T_1 = 300 \mathrm{K}$$

$$p_1 = 1.013 \times 10^5 \, \mathrm{Pa}$$

氮气为双原子气体由表查得　$\gamma = 1.40$

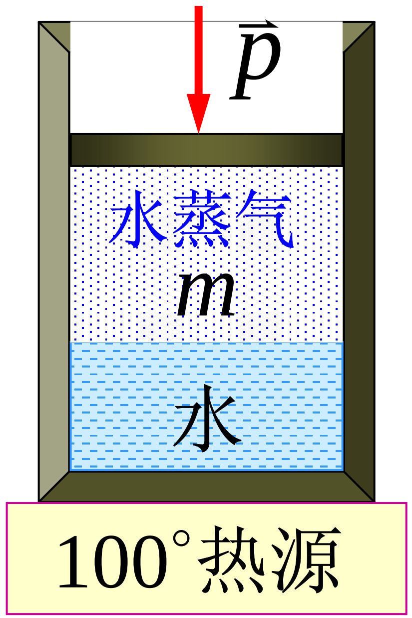
$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(\gamma - 1)/\gamma} = 98.0 \mathrm{K}$$

例3　在一气缸内放有一定量的水，活塞与汽缸间的摩擦不计缸壁由良导热材料制成. 作用于活塞上的压强 $p = 1.013\times10^{5}\,\mathrm{Pa}$. 开始时, 活塞与水面接触. 若使环境 (热源) 温度非常缓慢地升高到 100°C. 求把单位质量的水汽化为水蒸汽 , 水的内能改变了多少?

已知水的汽化热为 $L = 2.26\times10^{6}\,\mathrm{J\cdot kg^{-1}}$

水的密度 $\rho_{水} = 1040\,\mathrm{kg\cdot m^{-3}}$

水蒸汽的密度 $\rho_{蒸汽} = 0.598\,\mathrm{kg\cdot m^{-3}}$



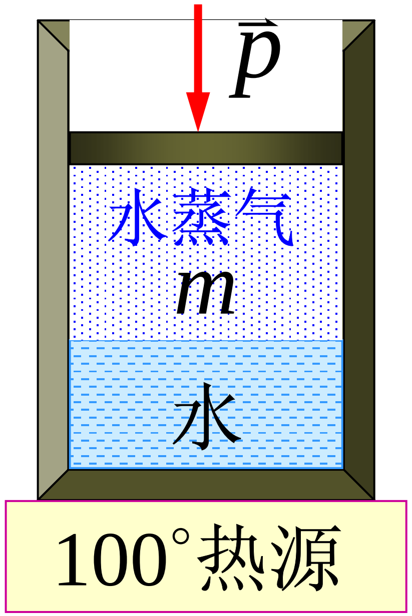
解　水汽化所需的热量 $Q = mL$

水汽化后体积膨胀为

$$\Delta V = m\left(\frac{1}{\rho_{蒸汽}} - \frac{1}{\rho_{水}}\right)$$

$$L = 2.26\times10^{6}\,\mathrm{J\cdot kg^{-1}} \qquad \rho_{水} = 1040\,\mathrm{kg\cdot m^{-3}}$$

$$\rho_{蒸汽} = 0.598\,\mathrm{kg\cdot m^{-3}}$$



$$\Delta V = m\left(\frac{1}{\rho_{蒸汽}} - \frac{1}{\rho_{水}}\right)$$

$$W = \int p\mathrm{d}V = p\Delta V = pm\left(\frac{1}{\rho_{蒸汽}} - \frac{1}{\rho_{水}}\right)$$

$$\Delta E = Q - W = mL - pm\left(\frac{1}{\rho_{蒸汽}} - \frac{1}{\rho_{水}}\right)$$

$$\frac{\Delta E}{m} = L - p\left(\frac{1}{\rho_{蒸汽}} - \frac{1}{\rho_{水}}\right) = 2.09\times10^{6}\,\mathrm{J\cdot kg^{-1}}$$