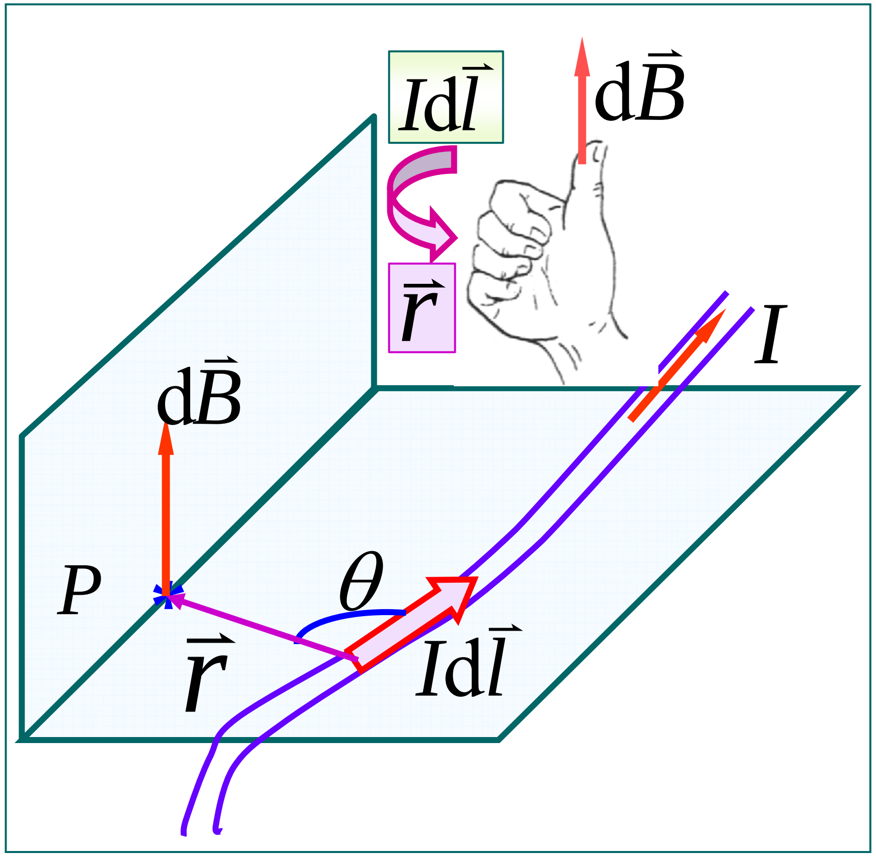
一　毕奥—萨伐尔定律

(电流元在空间产生的磁场)

$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}l\sin\theta}{r^2}$$

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l}\times\vec{r}}{r^3}$$

真空磁导率 $\mu_0 = 4\pi\times10^{-7}\,\mathrm{N\cdot A^{-2}}$



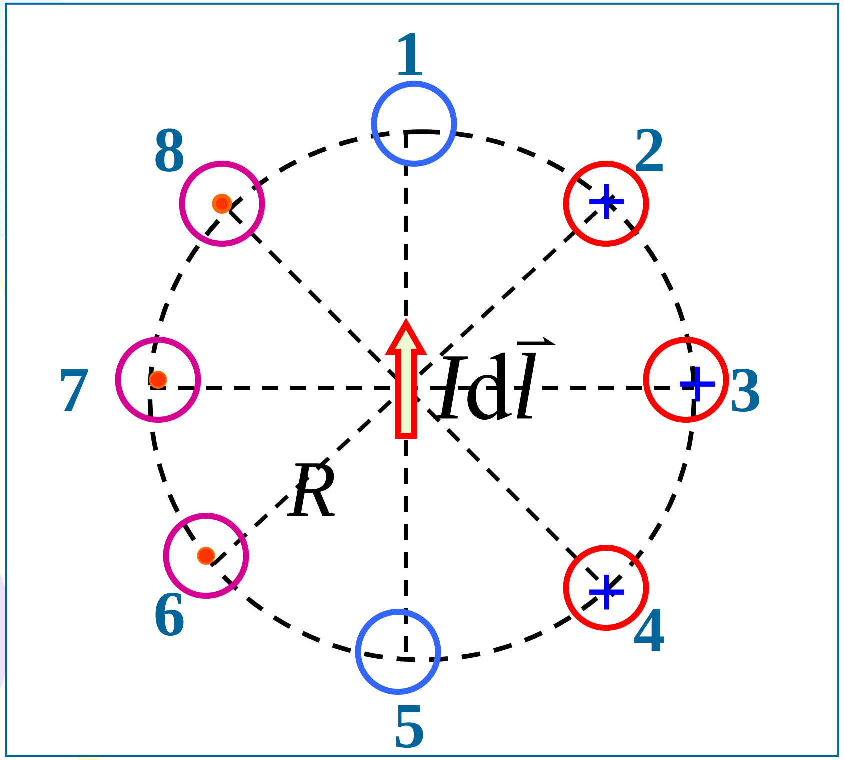
◆ 任意载流导线在点 P 处的磁感强度

磁感强度叠加原理　$$\vec{B} = \int \mathrm{d}\vec{B} = \int \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\vec{l}\times\vec{r}}{r^3}$$

$$\mathrm{d}\vec{B}=\frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l}\times\vec{r}}{r^3}$$

毕奥—萨伐尔定律

例　判断下列各点磁感强度的方向和大小.



1、5 点：$\mathrm{d}B=0$

3、7点：$\mathrm{d}B=\dfrac{\mu_0 I\mathrm{d}l}{4\pi R^2}$

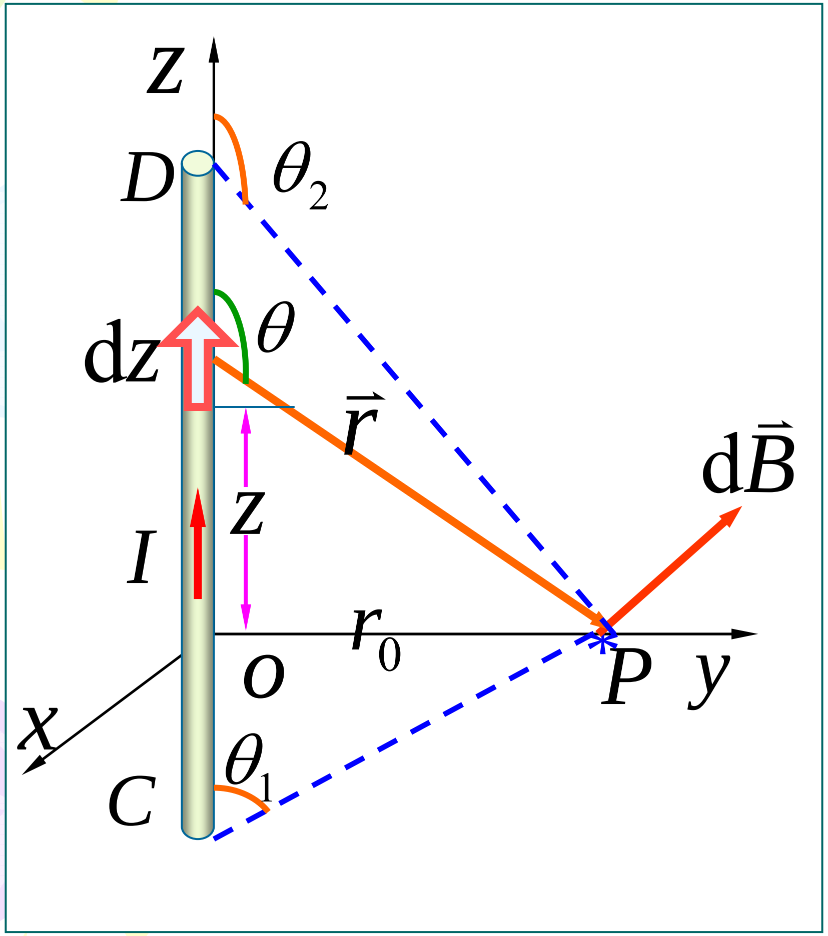
2、4、6、8 点：

$$\mathrm{d}B=\frac{\mu_0 I\mathrm{d}l}{4\pi R^2}\sin 45^0$$

二　毕奥---萨伐尔定律应用举例

$\mathrm{d}\vec{B}$ 方向均沿 x 轴的负方向

◆ **例1** 载流长直导线的磁场.



解　$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}z\sin\theta}{r^2}$$

$$B = \int \mathrm{d}B = \frac{\mu_0}{4\pi}\int_{CD}\frac{I\mathrm{d}z\sin\theta}{r^2}$$

$$z = -r_0\cot\theta, r = r_0/\sin\theta$$

$$\mathrm{d}z = r_0\mathrm{d}\theta/\sin^2\theta$$

$$B = \frac{\mu_0 I}{4\pi r_0}\int_{\theta_1}^{\theta_2}\sin\theta\mathrm{d}\theta$$

$$B=\frac{\mu_0 I}{4\pi\ r_0}\int_{\theta_1}^{\theta_2}\sin\theta \mathrm{d}\theta=\frac{\mu_0 I}{4\pi\ r_0}(\cos\theta_1-\cos\theta_2)$$

$\vec{B}$ 的方向沿 x 轴的负方向.

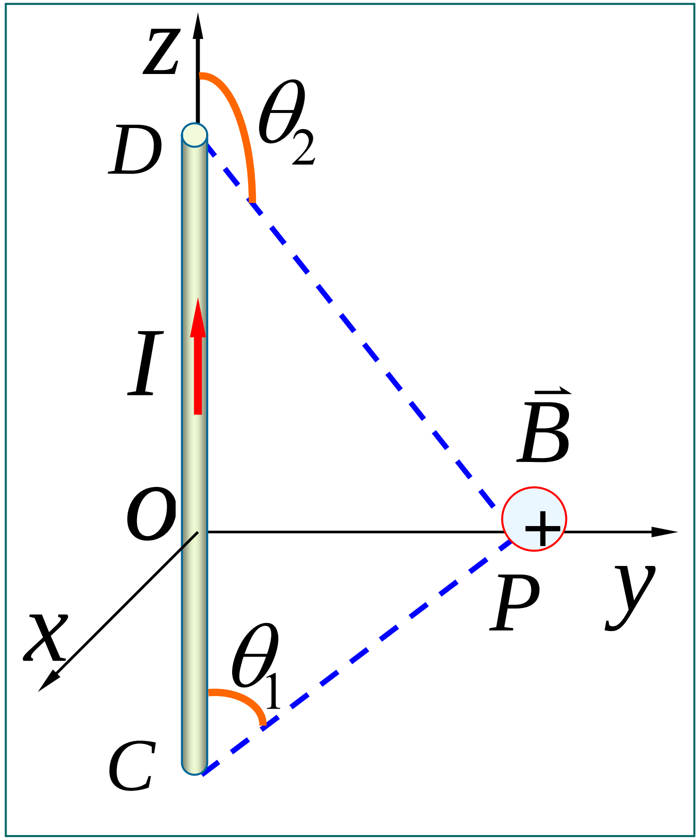
无限长载流长直导线的磁场.

$$B=\frac{\mu_0 I}{4\pi\ r_0}(\cos\theta_1-\cos\theta_2)$$

$\theta_1\to 0$

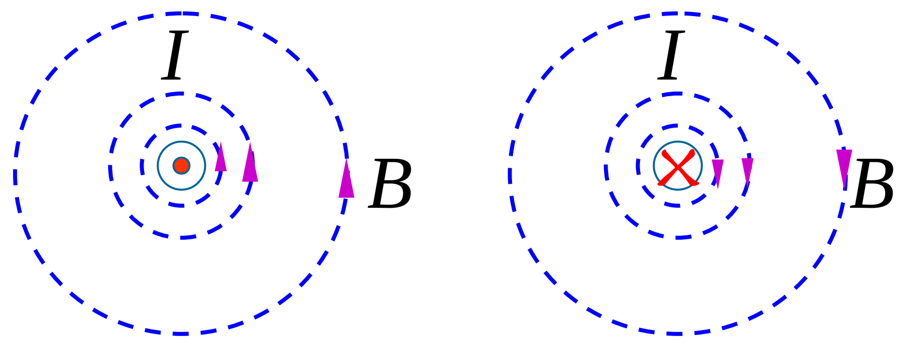
$\theta_2\to\pi$

$$B=\frac{\mu_0 I}{2\pi\ r_0}$$



◆ 无限长载流长直导线的磁场

$$B=\frac{\mu_0 I}{2\pi r}$$



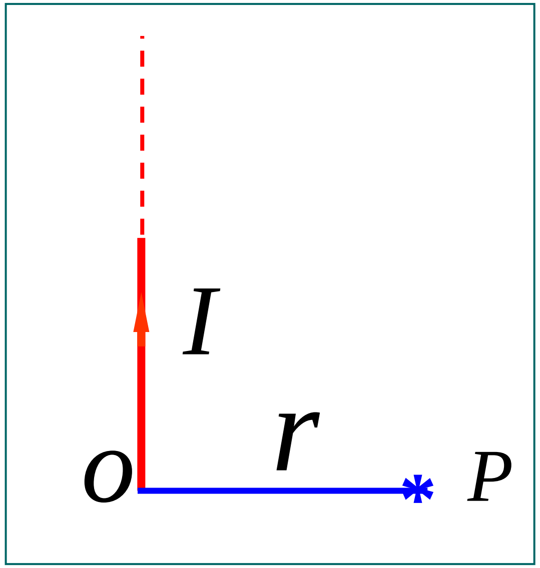
◆ 电流与磁感强度成右螺旋关系

半无限长载流长直导线的磁场

$$\theta_1 \to \frac{\pi}{2}$$

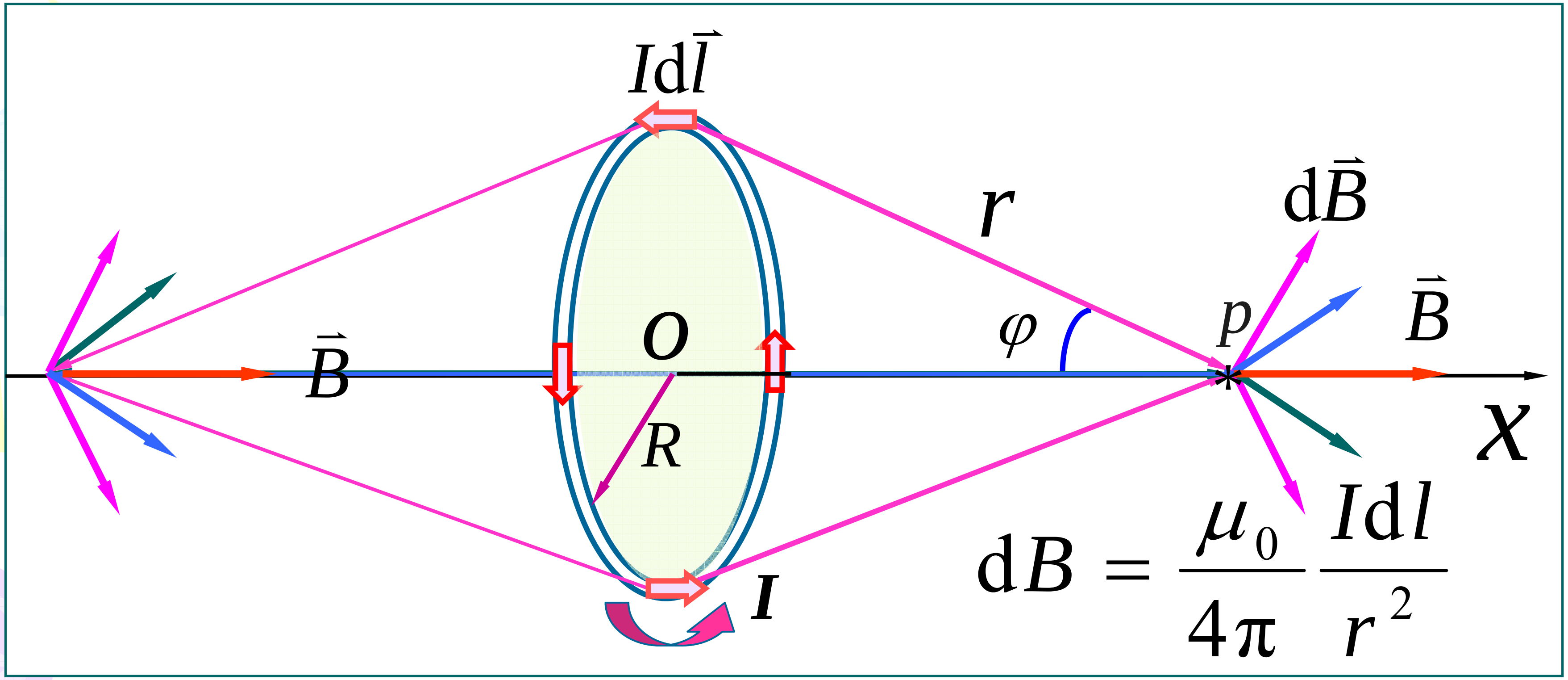
$$\theta_2 \to \pi$$

$$B_P=\frac{\mu_0 I}{4\pi r}$$

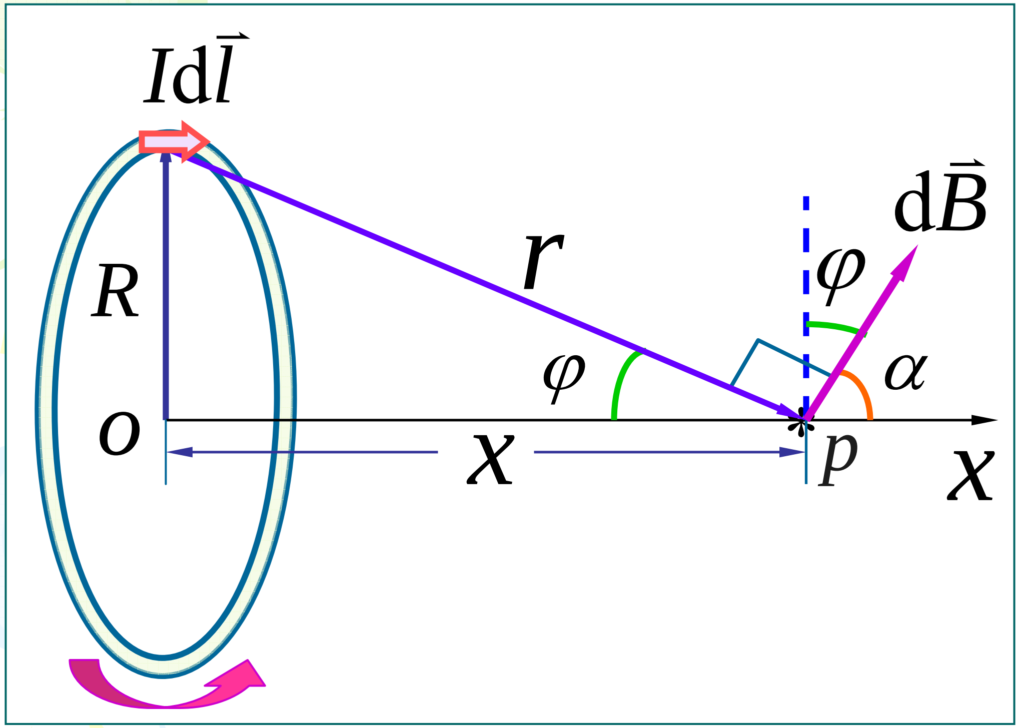


例2　圆形载流导线的磁场.

真空中，半径为R 的载流导线，通有电流I，称圆电流. 求其轴线上一点 p 的磁感强度的方向和大小.



解　根据对称性分析　　$B=B_x=\int \mathrm{d}B\sin\varphi$



$$\cos\alpha = R/r$$

$$r^2 = R^2 + x^2$$

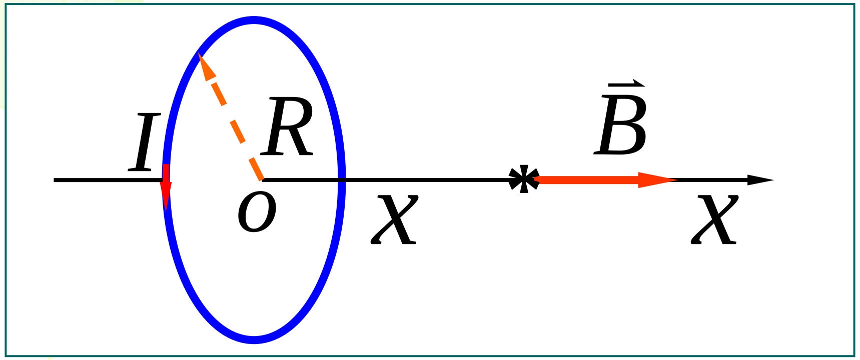
$$B = \frac{\mu_0 I}{4\pi}\int_l \frac{\cos\alpha \mathrm{d}l}{r^2}$$

$$\mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}l}{r^2}$$

$$B = \frac{\mu_0 I R}{4\pi r^3}\int_0^{2\pi R}\mathrm{d}l$$

$$\mathrm{d}B_x = \frac{\mu_0}{4\pi}\frac{I\cos\alpha \mathrm{d}l}{r^2}$$

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$



$$B=\frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

讨论

1）若线圈有 N 匝

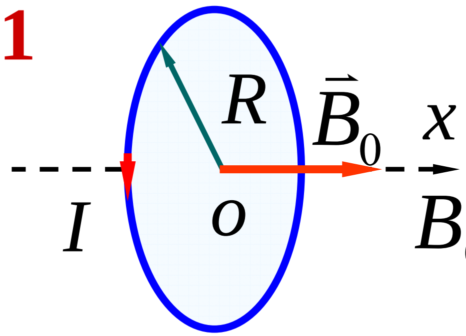
$$B=\frac{N\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

2）$x<0$　$\vec{B}$ 的方向不变(I 和 $\vec{B}$ 成右螺旋关系）

3）$x=0$　$$B=\frac{\mu_0 I}{2R}$$

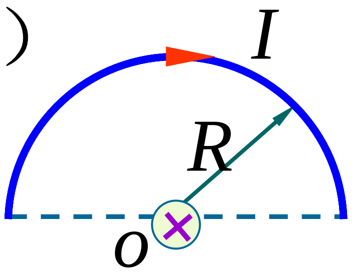
4）$x>>R$　$$B=\frac{\mu_0 IR^2}{2x^3},\qquad B=\frac{\mu_0 IS}{2\pi x^3}$$

（1）



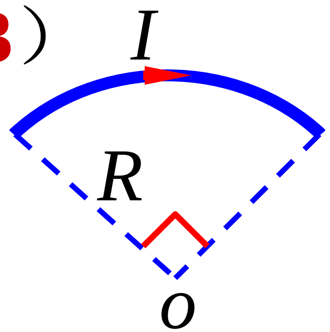
$$B_0 = \frac{\mu_0 I}{2R}$$

（2）



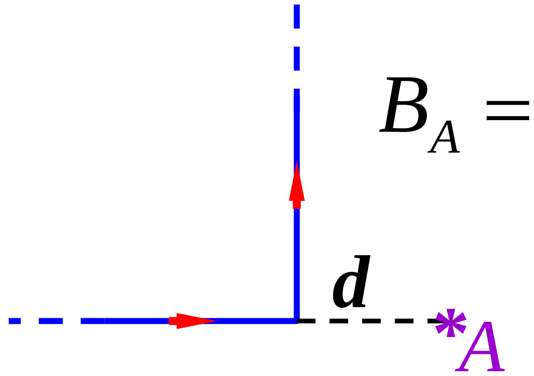
$$B_0 = \frac{\mu_0 I}{4R}$$

（3）



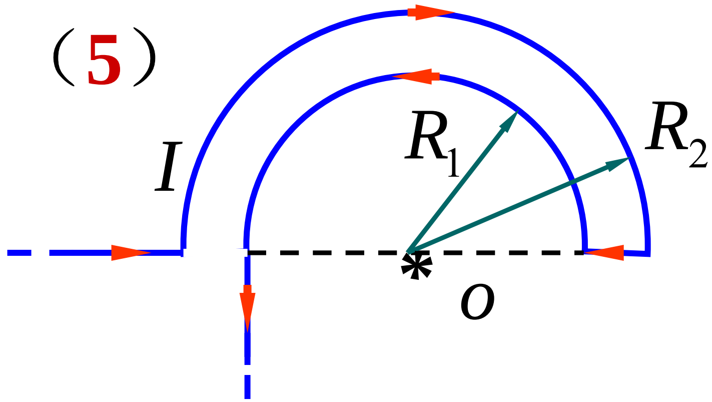
$$B_0 = \frac{\mu_0 I}{8R}$$

（4）



$$B_A = \frac{\mu_0 I}{4\pi d}$$

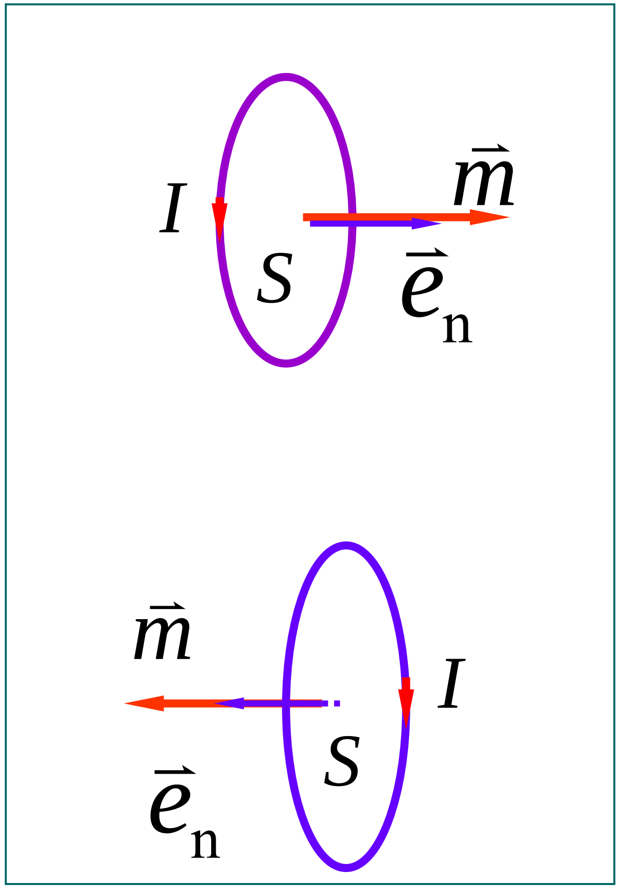
（5）



$$B_0 = \frac{\mu_0 I}{4R_2} - \frac{\mu_0 I}{4R_1} - \frac{\mu_0 I}{4\pi R_1}$$

三　磁偶极矩

$$\vec{m} = IS\,\vec{e}_{\mathrm{n}}$$



例2中圆电流磁感强度公式也可写成

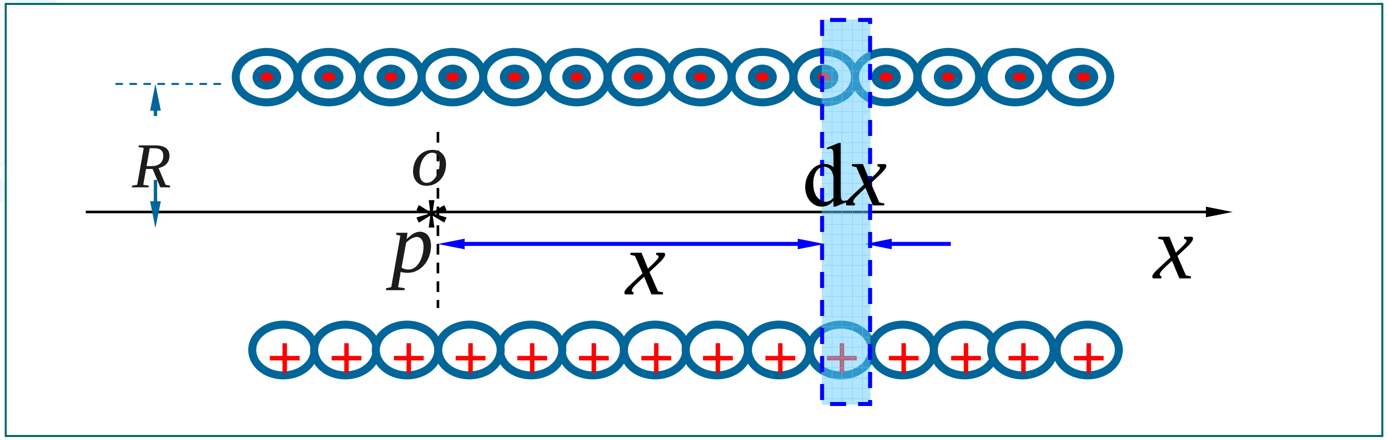
$$B = \frac{\mu_0 I R^2}{2x^3} \qquad \vec{B} = \frac{\mu_0 \vec{m}}{2\pi\, x^3}$$

$$\vec{B} = \frac{\mu_0 m}{2\pi\, x^3}\vec{e}_{\mathrm{n}}$$

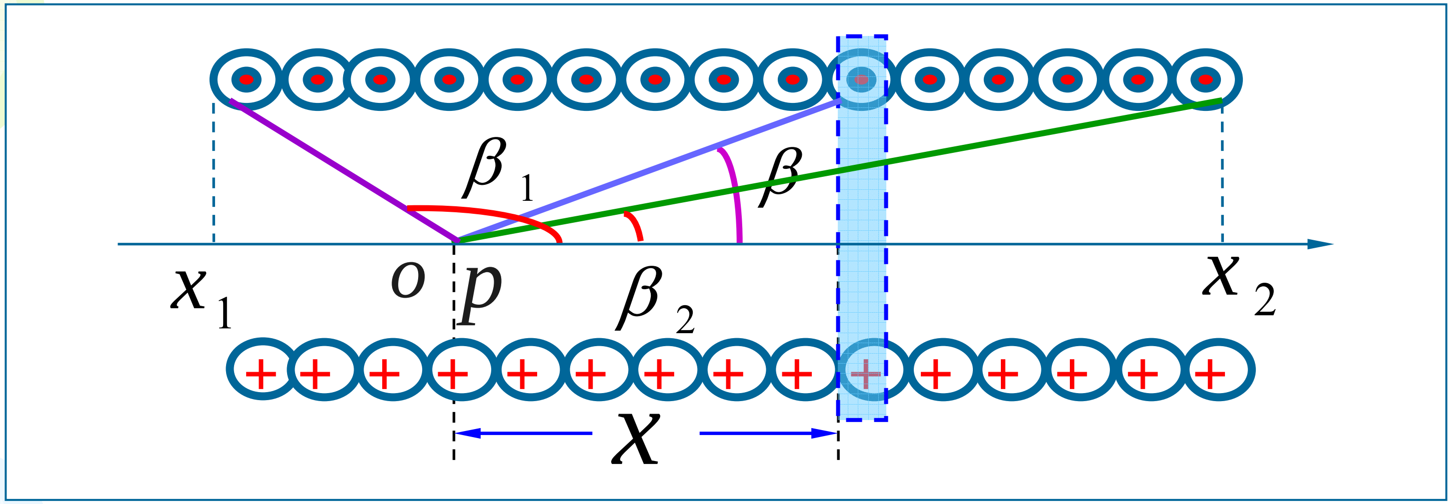
说明：只有当圆形电流的面积S很小，或场点距圆电流很远时，才能把圆电流叫做磁偶极子.

例3　载流直螺线管的磁场

如图所示，有一长为l，半径为R的载流密绕直螺线管，螺线管的总匝数为N，通有电流I. 设把螺线管放在真空中，求管内轴线上一点处的磁感强度.



解　由圆形电流磁场公式 $B=\dfrac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$



$$\mathrm{d}B = \frac{\mu_0}{2}\frac{R^2 In\,\mathrm{d}x}{\left(R^2 + x^2\right)^{3/2}}$$

$$x = R\cot\beta$$

$$\mathrm{d}x = -R\csc^2\beta\mathrm{d}\beta$$

$$B = \int \mathrm{d}B = \frac{\mu_0 nI}{2}\int_{x_1}^{x_2}\frac{R^2\mathrm{d}x}{\left(R^2 + x^2\right)^{3/2}}$$

$$R^2 + x^2 = R^2\csc^2\beta$$

$$B = -\frac{\mu_0 nI}{2}\int_{\beta_1}^{\beta_2}\frac{R^3\csc^2\beta\mathrm{d}\beta}{R^3\csc^3\beta\mathrm{d}\beta} = -\frac{\mu_0 nI}{2}\int_{\beta_1}^{\beta_2}\sin\beta\,\mathrm{d}\beta$$

讨 论

$$B=\frac{\mu_0 nI}{2}\left(\cos\beta_2-\cos\beta_1\right)$$

（**1**）***P***点位于管内轴线中点　　$\beta_1=\pi-\beta_2$

$$\cos\beta_1=-\cos\beta_2 \qquad \cos\beta_2=\frac{l/2}{\sqrt{(l/2)^2+R^2}}$$

$$B=\mu_0 nI\cos\beta_2=\frac{\mu_0 nI}{2}\frac{l}{\left(l^2/4+R^2\right)^{1/2}}$$

若　$l>>R$　　$B=\mu_0 nI$

(2) 无限长的螺线管

$$B = \mu_0 n I$$

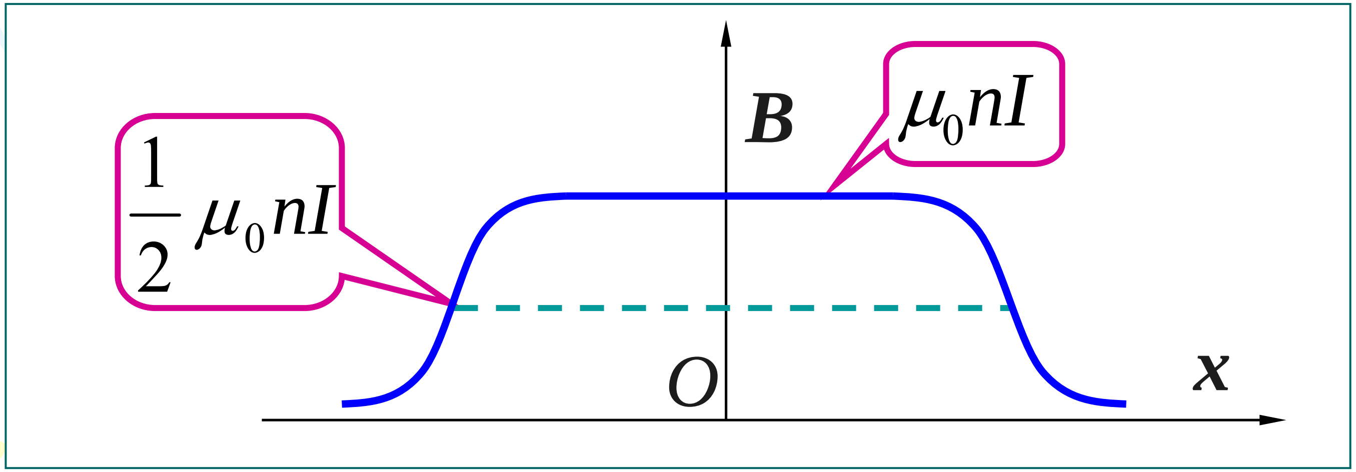
或由 $\beta_1 = \pi, \beta_2 = 0$ 代入

$$B = \frac{\mu_0 n I}{2}(\cos\beta_2 - \cos\beta_1)$$

（3）半无限长螺线管

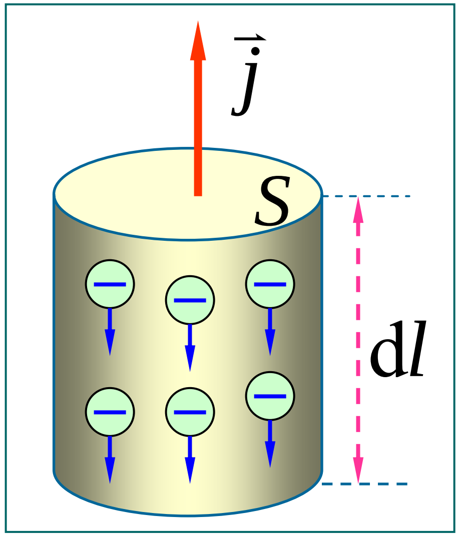
$$\beta_1 = \frac{\pi}{2}, \beta_2 = 0$$

$$B = \frac{1}{2}\mu_0 n I$$



四　运动电荷的磁场

毕— 萨定律　$\mathrm{d}\vec{B}=\dfrac{\mu_0}{4\pi}\dfrac{I\mathrm{d}\vec{l}\times\vec{r}}{r^3}$



$$I\mathrm{d}\vec{l}=\vec{j}S\mathrm{d}l=nS\mathrm{d}lq\vec{v}$$

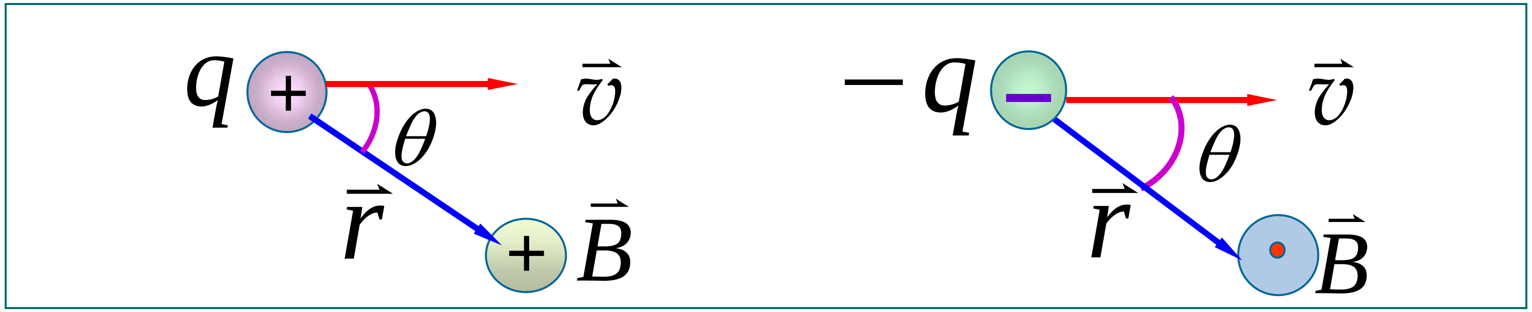
$$\mathrm{d}\vec{B}=\frac{\mu_0}{4\pi}\frac{nS\mathrm{d}lq\vec{v}\times\vec{r}}{r^3}$$

$$\mathrm{d}N=nS\,\mathrm{d}l$$

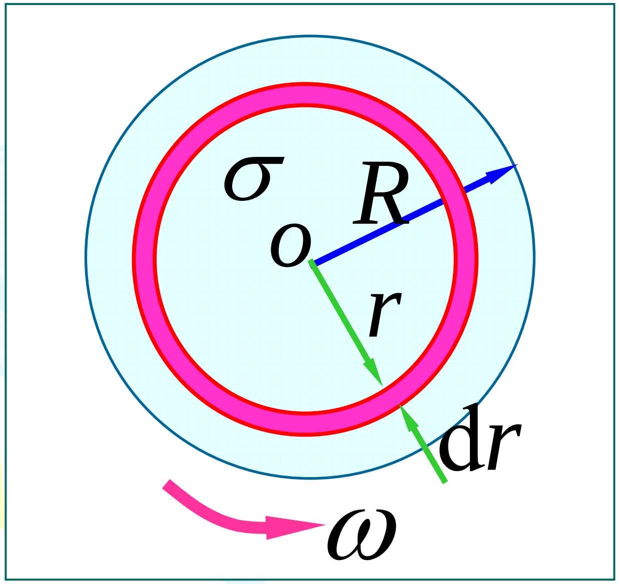
运动电荷的磁场

$$\vec{B}=\frac{\mathrm{d}\vec{B}}{\mathrm{d}N}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

实用条件　$v<<c$



例4　半径 为 R 的带电薄圆盘的电荷面密度为 σ, 并以角速度 ω 绕通过盘心垂直于盘面的轴转动，求圆盘中心的磁感强度.



解法一　圆电流的磁场

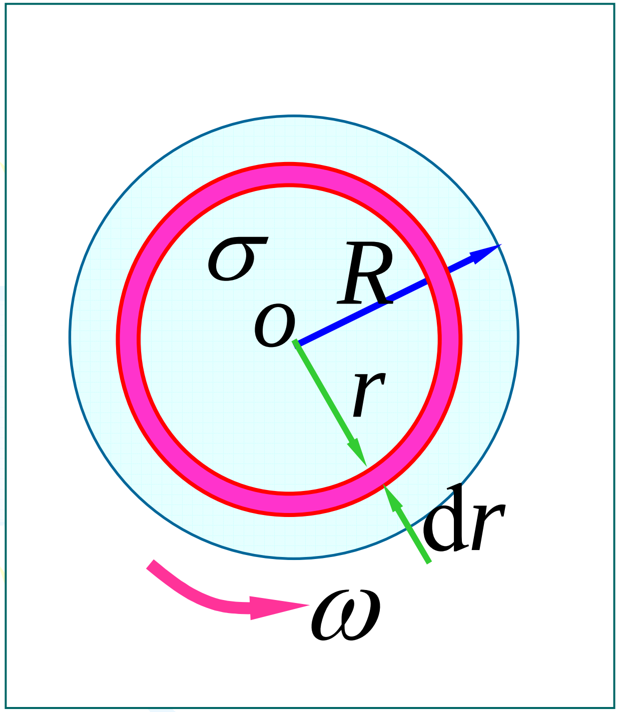
$$\mathrm{d}I=\frac{\omega}{2\pi}\sigma 2\pi\ r\mathrm{d}r=\sigma\omega r\mathrm{d}r$$

$$\mathrm{d}B=\frac{\mu_0\mathrm{d}I}{2r}=\frac{\mu_0\sigma\omega}{2}\mathrm{d}r$$

$$B=\frac{\mu_0\sigma\omega}{2}\int_0^R\mathrm{d}r=\frac{\mu_0\sigma\omega R}{2}$$

$\sigma>0,\ \vec{B}$ 向外

$\sigma<0,\ \vec{B}$ 向内



解法二　运动电荷的磁场

$$\mathrm{d}B_0 = \frac{\mu_0}{4\pi}\frac{\mathrm{d}qv}{r^2}$$

$$\mathrm{d}q = \sigma 2\pi\, r\mathrm{d}r$$

$$\mathrm{d}B = \frac{\mu_0 \sigma \omega}{2}\mathrm{d}r$$

$$v = \omega r$$

$$B = \frac{\mu_0 \sigma \omega}{2}\int_0^R \mathrm{d}r = \frac{\mu_0 \sigma \omega R}{2}$$