一　静电场

实验证实了两静止电荷间存在相互作用的静电力，但其相互作用是怎样实现的？

电荷 ⟷ 电场 ⟷ 电荷

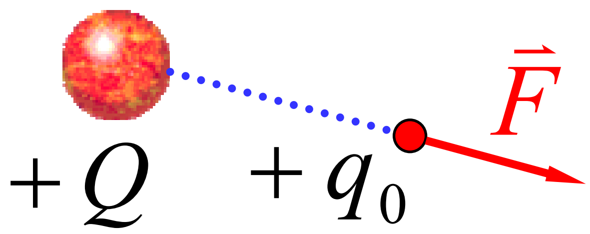
场是一种特殊形态的物质

场
实物
} 物质

二　电场强度

$$\vec{E} = \frac{\vec{F}}{q_0}$$

电场中某点处的电场强度 $\vec{E}$ 等于位于该点处的单位试验电荷所受的力，其方向为正电荷受力方向.



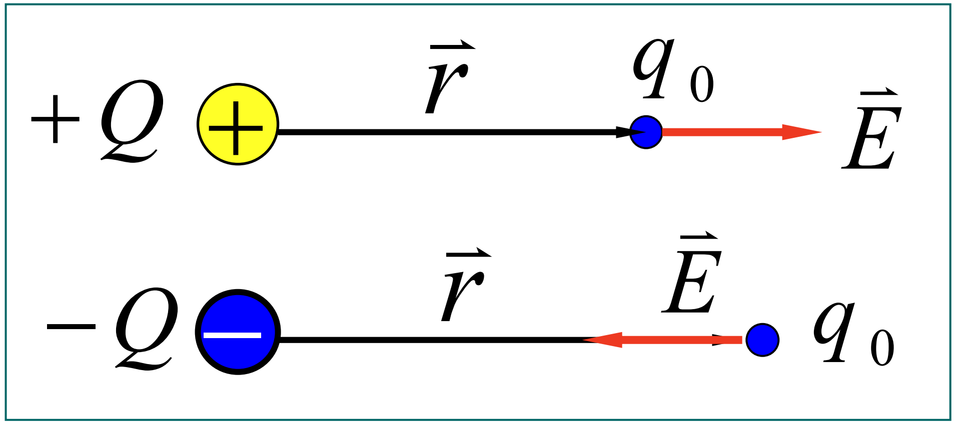
$+Q$：场源电荷
$+q_0$：试验电荷

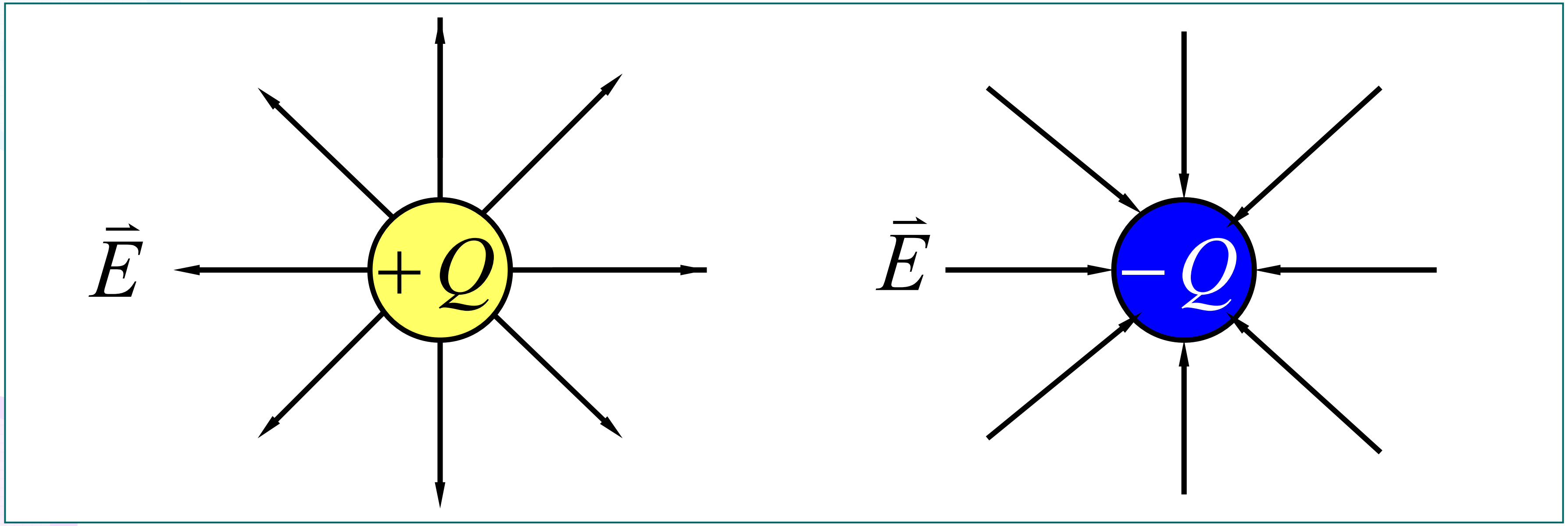
（试验电荷为点电荷、且足够小，故对原电场几乎无影响）

- 单位　$\mathrm{N \cdot C^{-1}}$　$\mathrm{V \cdot m^{-1}}$
- 电荷 q 在电场中受力　$\vec{F} = q\vec{E}$

三 点电荷的电场强度

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$$





$$r \to 0 \quad E \to \infty ?$$

例 把一个点电荷（$q=-62\times10^{-9}\text{C}$）放在电场中某点处，该电荷受到的电场力为$\vec{F}=3.2\times10^{-6}\vec{i}+1.3\times10^{-6}\vec{j}\text{N}$，求该电荷所在处的电场强度.

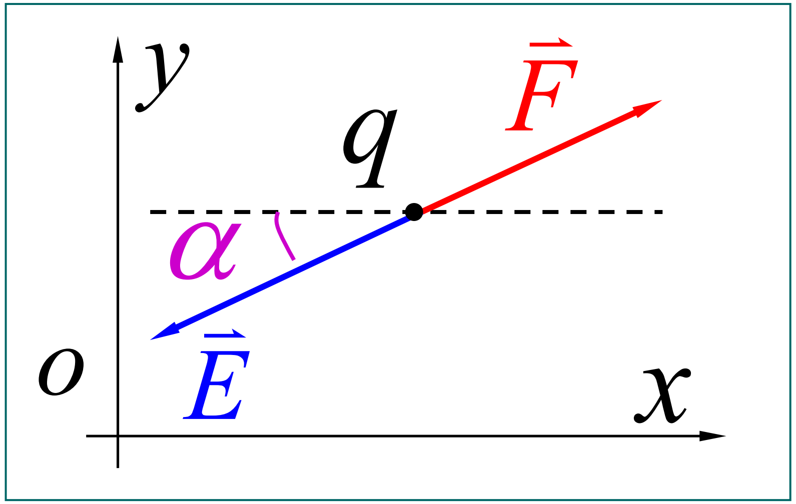
解 $$\vec{E}=\frac{\vec{F}}{q}=-(51.6\vec{i}+21.0\vec{j})\text{N}\cdot\text{C}^{-1}$$

大小 $$|\vec{E}|=E=\sqrt{(-51.6)^2+(-21.0)^2}\text{N}\cdot\text{C}^{-1}$$

$$=55.71\text{N}\cdot\text{C}^{-1}$$

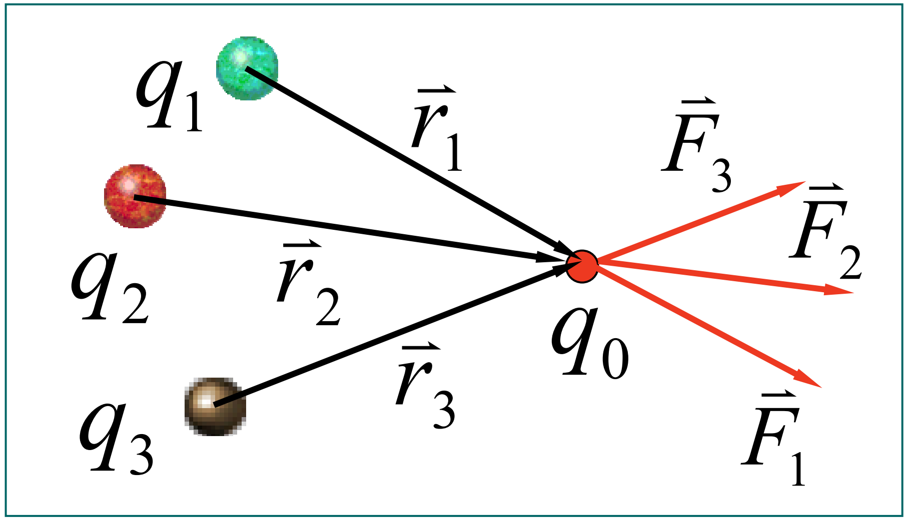
方向 $$\alpha=\arctan\frac{E_y}{E_x}$$

$$=22.1^\circ$$



四　电场强度的叠加原理

点电荷 q_i 对 q_0 的作用力



$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^3}\vec{r}_i$$

由力的叠加原理得 q_0 所受合力 $\vec{F} = \sum_i \vec{F}_i$

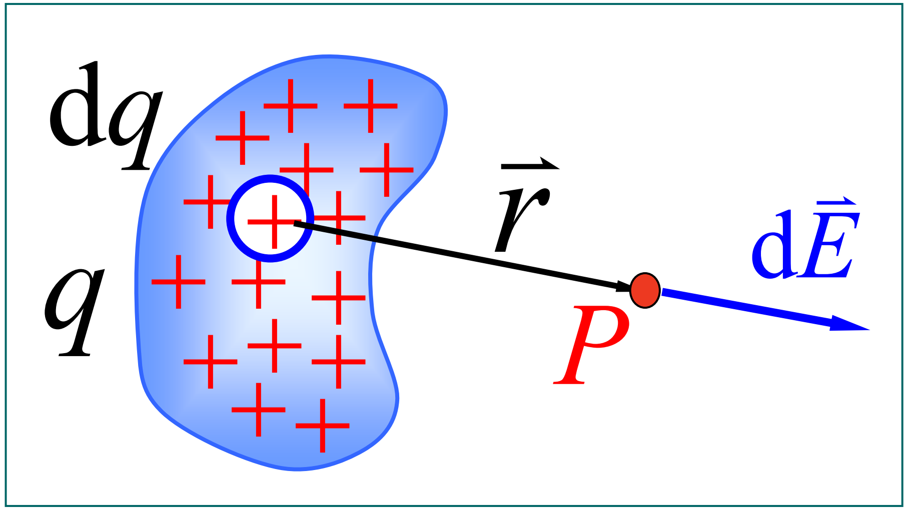
故 q_0 处总电场强度 $\vec{E} = \dfrac{\vec{F}}{q_0} = \sum_i \dfrac{\vec{F}_i}{q_0}$

电场强度的叠加原理

$$\vec{E} = \sum_i \vec{E}_i$$

◆ 电荷连续分布情况

$$\mathrm{d}\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r^2}\vec{e}_r$$



$$\vec{E}=\int\mathrm{d}\vec{E}=\int\frac{1}{4\pi\varepsilon_0}\frac{\vec{e}_r}{r^2}\mathrm{d}q$$

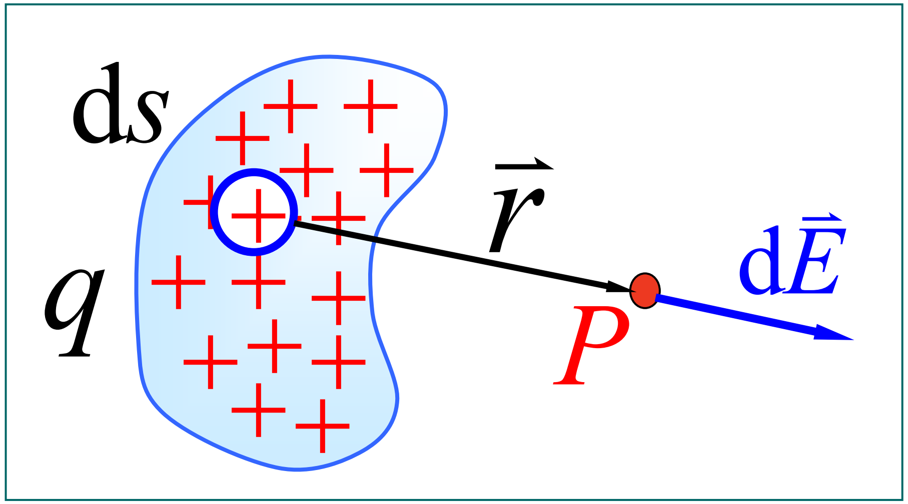
电荷体密度 $\rho=\dfrac{\mathrm{d}q}{\mathrm{d}V}$

点 P 处电场强度

$$\vec{E}=\int_V\frac{1}{4\pi\varepsilon_0}\frac{\rho\vec{e}_r}{r^2}\mathrm{d}V$$

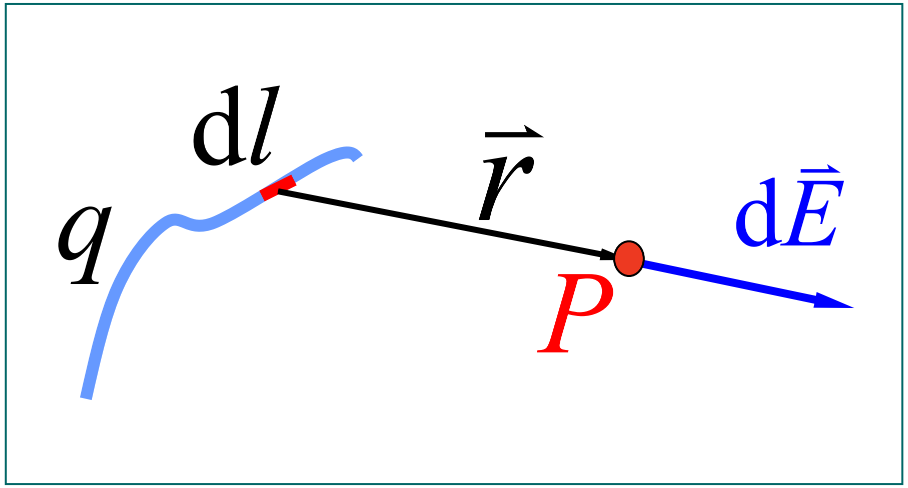
电荷面密度 $\sigma = \dfrac{\mathrm{d}q}{\mathrm{d}s}$

$$\vec{E} = \int_S \frac{1}{4\pi\varepsilon_0}\frac{\sigma\,\vec{e}_r}{r^2}\mathrm{d}s$$



电荷线密度 $\lambda = \dfrac{\mathrm{d}q}{\mathrm{d}l}$

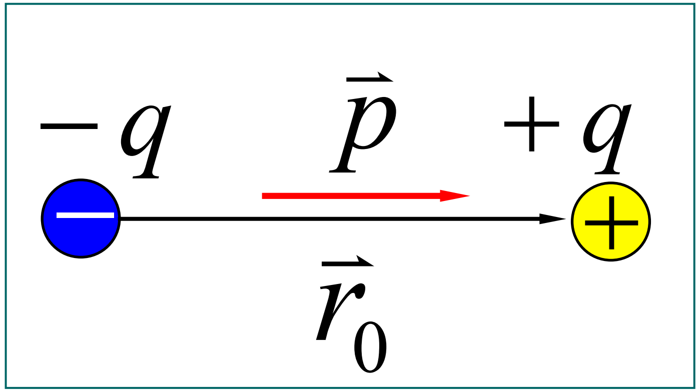
$$\vec{E} = \int_l \frac{1}{4\pi\varepsilon_0}\frac{\lambda\vec{e}_r}{r^2}\mathrm{d}l$$



五 电偶极子的电场强度

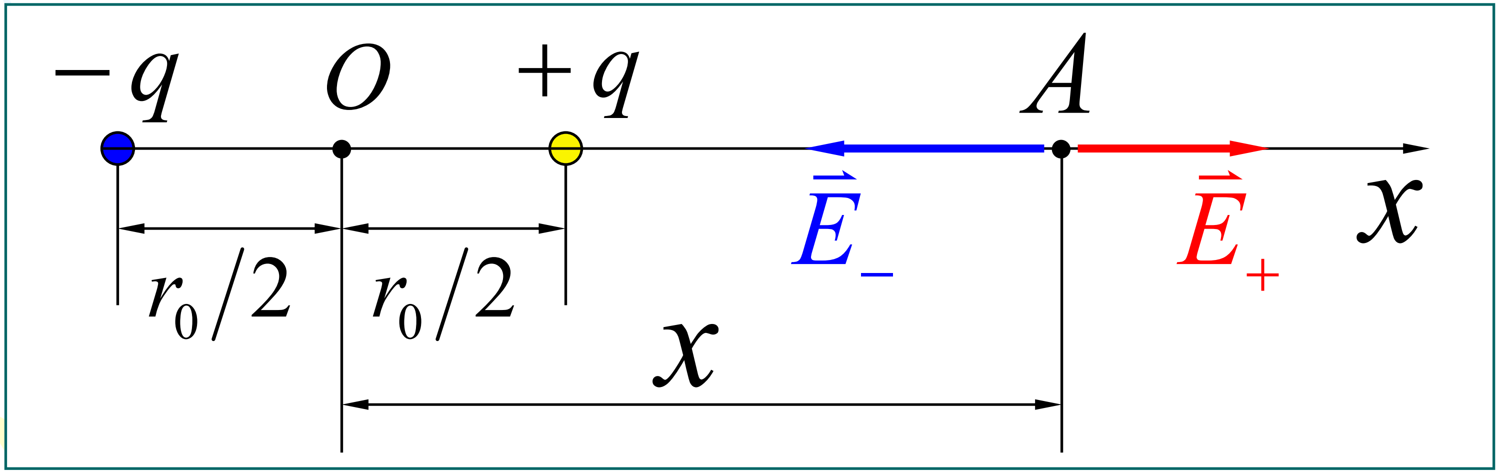
电偶极子的轴 $\vec{r}_0$

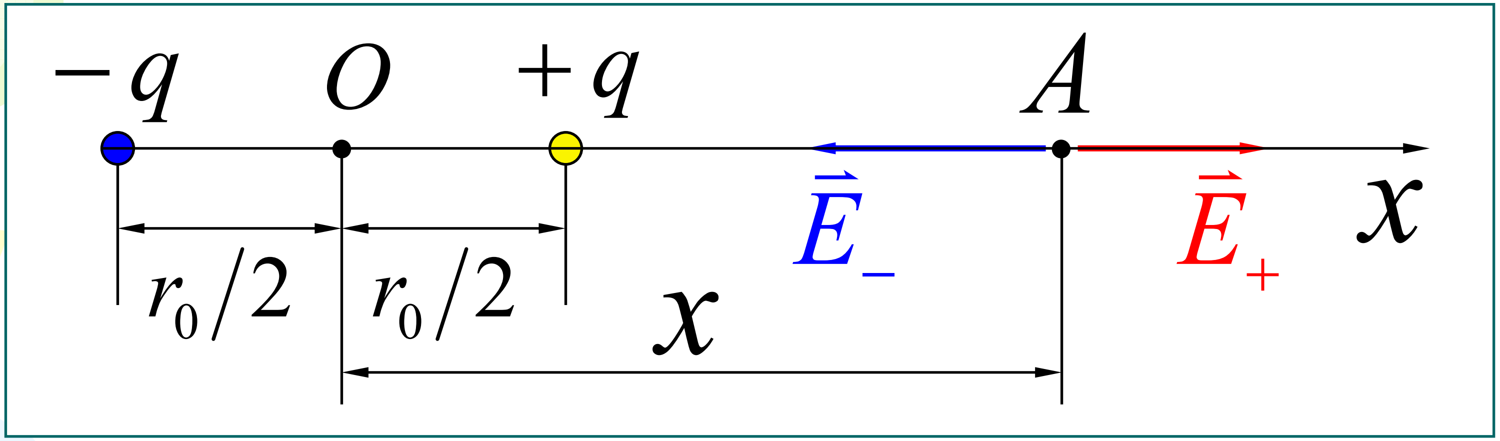
电偶极矩（电矩）$\vec{p}=q\vec{r}_0$



讨论

（1）电偶极子轴线延长线上一点的电场强度





$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x - r_0/2)^2}\vec{i} \qquad \vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(x + r_0/2)^2}\vec{i}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\varepsilon_0}\left[\frac{2xr_0}{(x^2 - r_0^2/4)^2}\right]\vec{i}$$

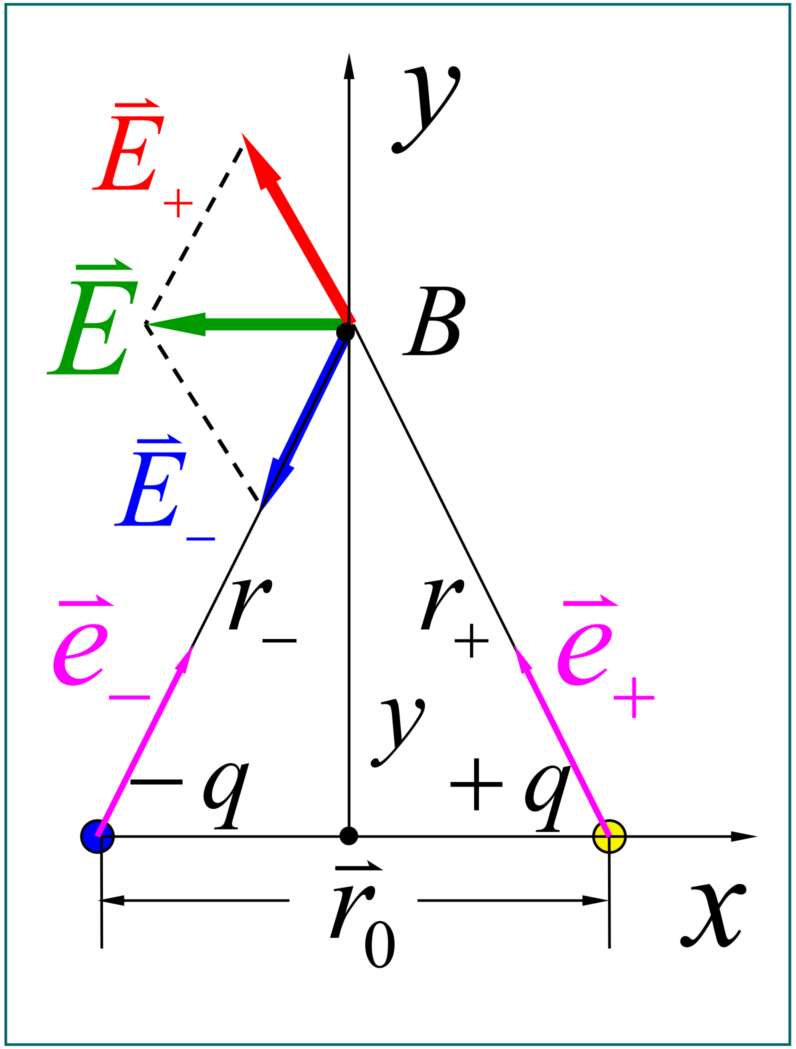
$x >> r_0$ 　 $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2r_0 q}{x^3}\vec{i} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$$

（2）电偶极子轴线的中垂线上一点的电场强度

$$\begin{cases} \vec{E}_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+^2}\vec{e}_+ \\ \vec{E}_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-^2}\vec{e}_- \end{cases}$$

$$r_+ = r_- = r = \sqrt{y^2 + (\frac{r_0}{2})^2}$$

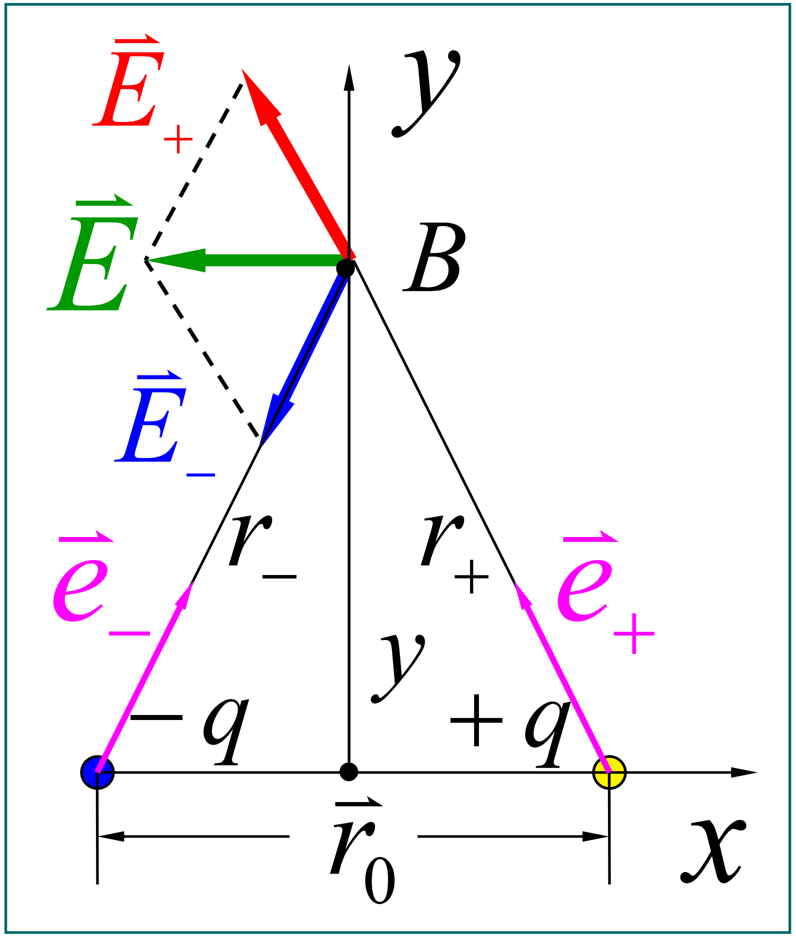
$$\begin{cases} \vec{e}_+ = (-r_0/2\vec{i} + y\vec{j})/r \\ \vec{e}_- = (r_0/2\vec{i} + y\vec{j})/r \end{cases}$$



$$\begin{cases} \vec{E}_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^3}(y\vec{j} - \dfrac{r_0}{2}\vec{i}) \\ \vec{E}_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^3}(y\vec{j} + \dfrac{r_0}{2}\vec{i}) \end{cases}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{qr_0\vec{i}}{r^3}$$

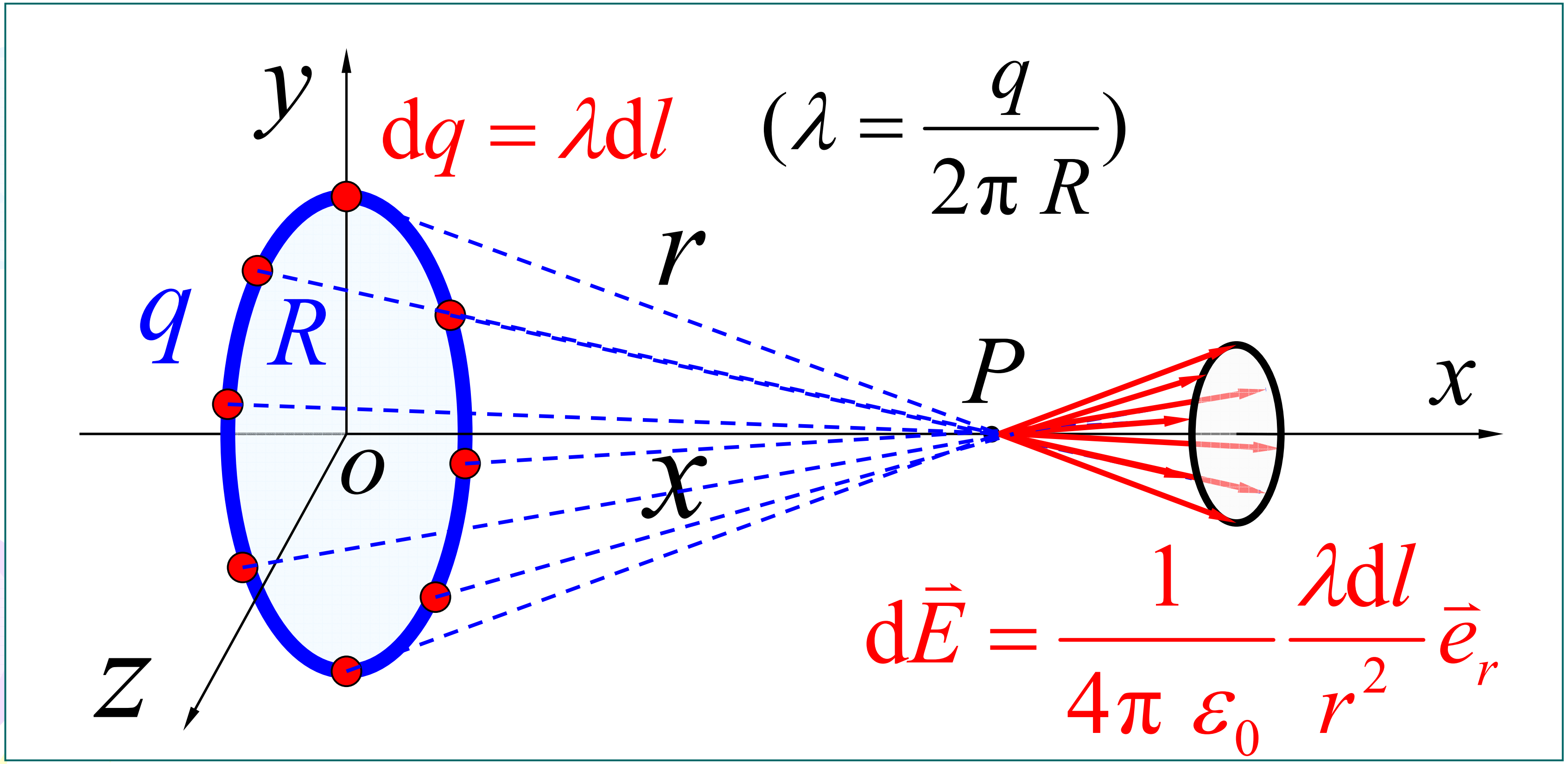
$$= -\frac{1}{4\pi\varepsilon_0}\frac{qr_0\vec{i}}{(y^2 + \frac{r_0^2}{4})^{3/2}}$$

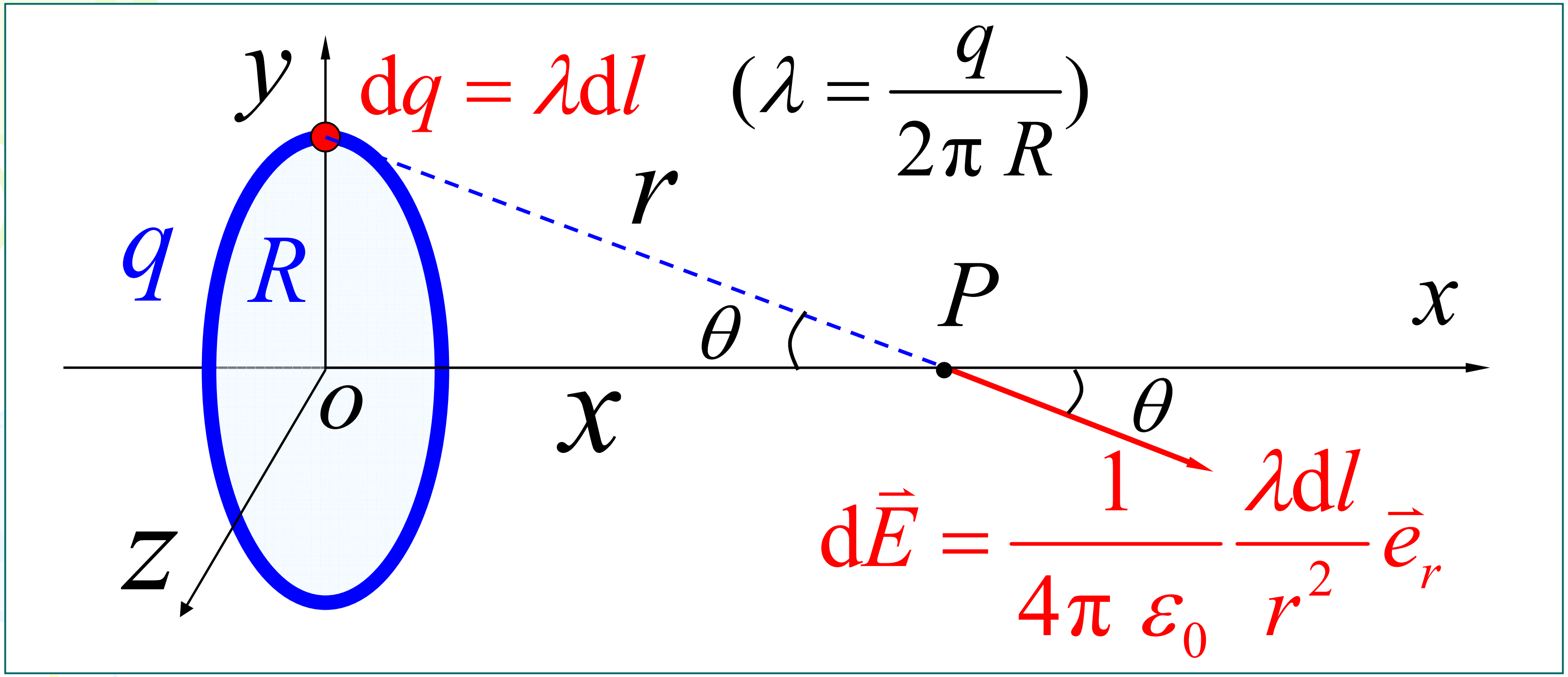


$y >> r_0$ $$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{qr_0\vec{i}}{y^3} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$$

例1　正电荷 q 均匀分布在半径为 R 的圆环上. 计算在环的轴线上任一点 P 的电场强度.

解　$\vec{E} = \int \mathrm{d}\vec{E}$　　由对称性有 $\vec{E} = E_x \vec{i}$





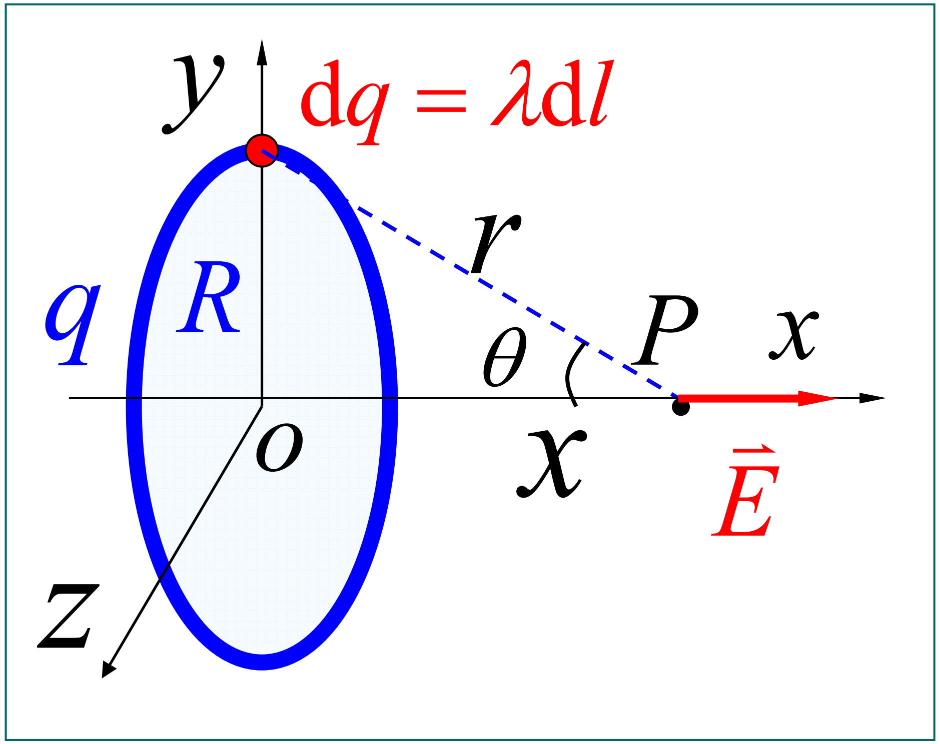
$$dq=\lambda dl \quad \left(\lambda=\frac{q}{2\pi R}\right)$$

$$d\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\vec{e}_r$$

$$E=\int_l dE_x=\int_l dE\cos\theta=\int\frac{\lambda dl}{4\pi\varepsilon_0 r^2}\cdot\frac{x}{r}$$

$$=\int_0^{2\pi R}\frac{x\lambda dl}{4\pi\varepsilon_0 r^3}=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$E=\frac{qx}{4\pi\,\varepsilon_0(x^2+R^2)^{3/2}}$$



讨论

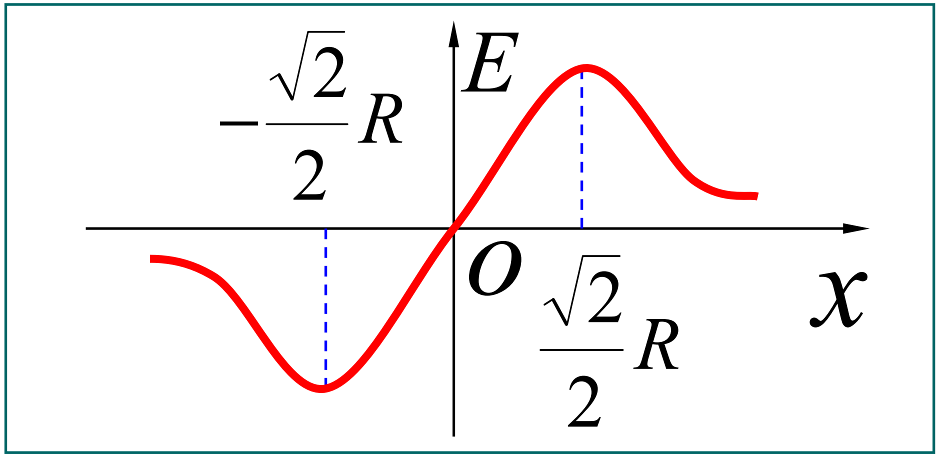
（1） $x>>R$

$$E\approx\frac{q}{4\pi\,\varepsilon_0 x^2}$$

（点电荷电场强度）

（2） $x\approx 0,\quad E_0\approx 0$

（3） $\frac{\mathrm{d}E}{\mathrm{d}x}=0,\quad x=\pm\frac{\sqrt{2}}{2}R$



例2　均匀带电薄圆盘轴线上的电场强度.

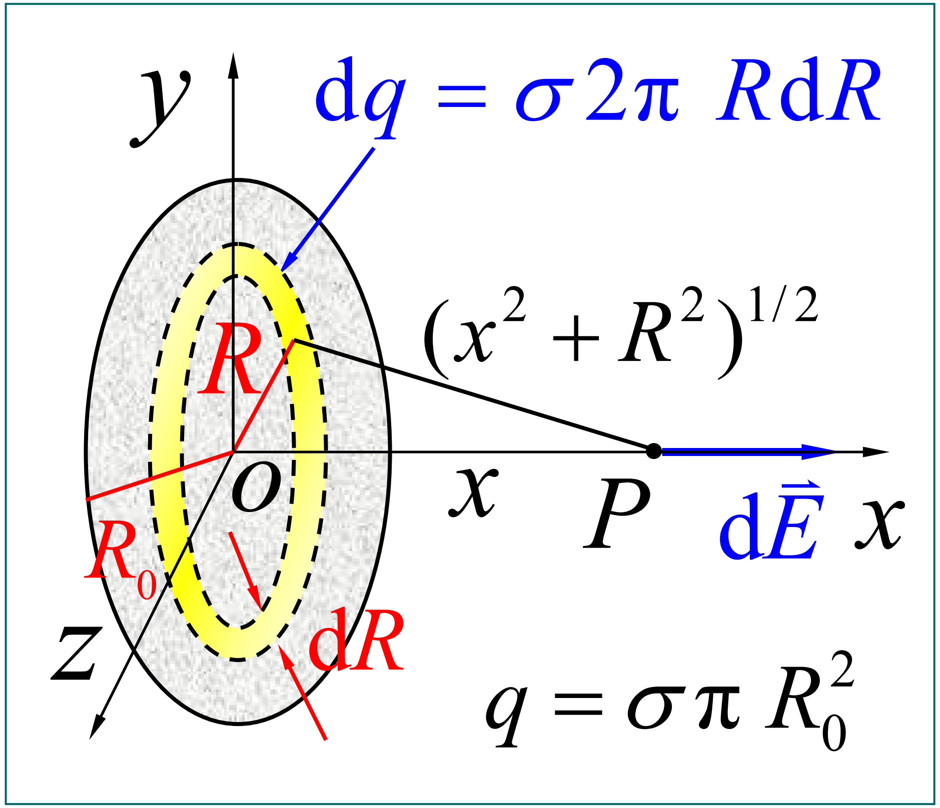
有一半径为 R_0, 电荷均匀分布的薄圆盘, 其电荷面密度为 σ． 求通过盘心且垂直盘面的轴线上任意一点处的电场强度.

解　由例 1

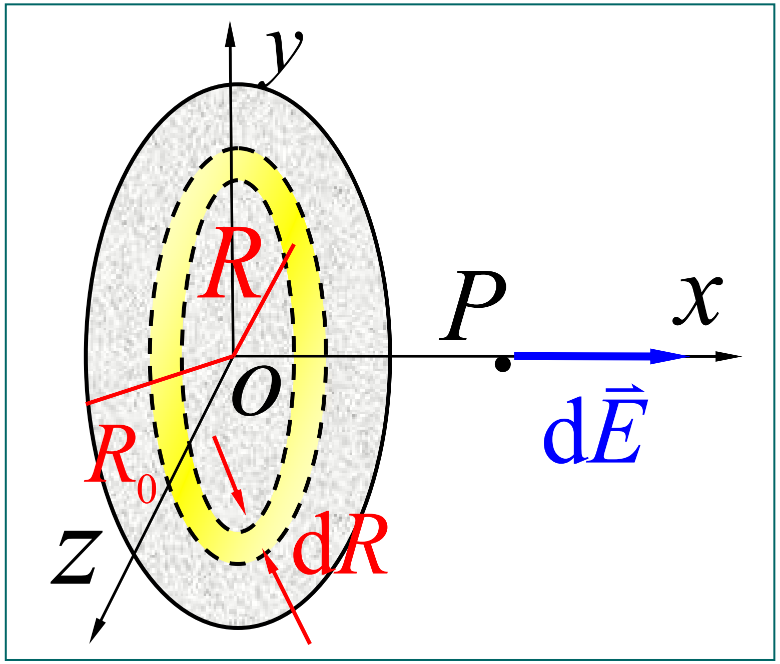
$$E=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$\mathrm{d}E_x=\frac{\mathrm{d}q\cdot x}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$=\frac{\sigma}{2\varepsilon_0}\frac{xR\mathrm{d}R}{(x^2+R^2)^{3/2}}$$



$$\mathrm{d}E_x = \frac{\sigma}{2\varepsilon_0}\frac{xR\mathrm{d}R}{(x^2+R^2)^{3/2}}$$



$$E = \int \mathrm{d}E_x$$

$$= \frac{\sigma x}{2\varepsilon_0}\int_0^{R_0}\frac{R\mathrm{d}R}{(x^2+R^2)^{3/2}}$$

$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x^2+R_0^2}}\right)$$

$$E=\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}}-\frac{1}{\sqrt{x^2+R_0^2}}\right)$$

讨论

$x << R_0$ $\quad E\approx\frac{\sigma}{2\varepsilon_0}$ 〔无限大均匀带电平面的电场强度〕

$x >> R_0$ $\quad E\approx\frac{q}{4\pi\varepsilon_0 x^2}$ （点电荷电场强度）

$$\left[(1+\frac{R_0^2}{x^2})^{-\frac{1}{2}}=1-\frac{1}{2}\cdot\frac{R_0^2}{x^2}+\cdots\right]$$