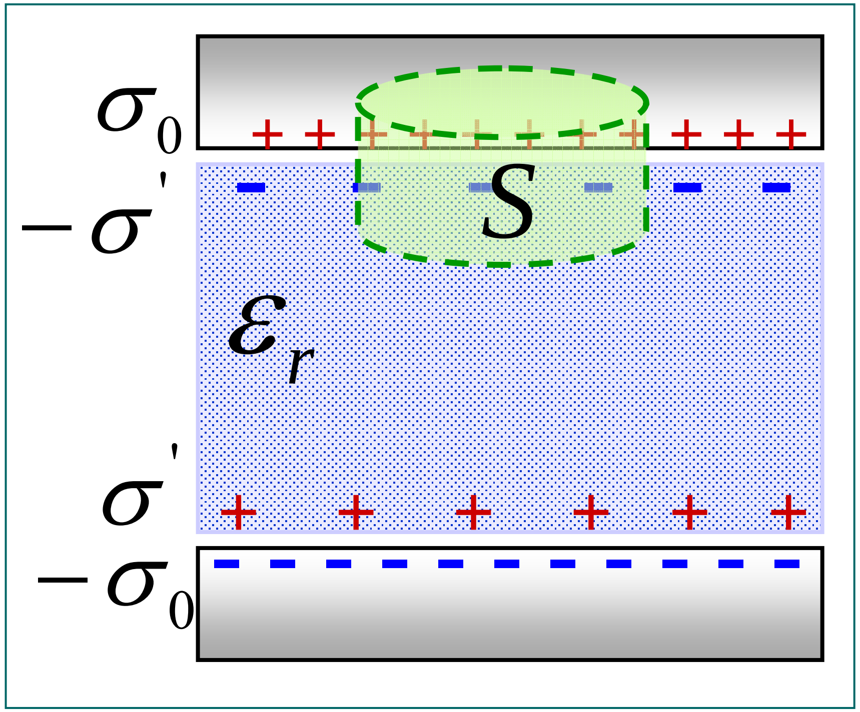
$$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}(Q_0 - Q')$$

$$Q' = \frac{\varepsilon_r - 1}{\varepsilon_r} Q_0$$

电容率 $\varepsilon = \varepsilon_0 \varepsilon_r$

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_0}{\varepsilon_0 \varepsilon_r}$$



电位移矢量 $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$ （均匀各相同性介质）

有介质时的高斯定理 $\oint_S \vec{D}\cdot d\vec{S} = \sum_i Q_{0i}$

电位移矢量

$$\boxed{\vec{D}=\vec{P}+\varepsilon_0\vec{E}}$$ （任何介质）

$$\boxed{\vec{D}=\varepsilon\vec{E}}$$ （均匀介质）

有介质时的高斯定理 $\oint_S \vec{D}\cdot \mathrm{d}\vec{S}=\sum_i Q_{0i}$

电容率 $\varepsilon=\varepsilon_0\varepsilon_r$

极化电荷面密度 $\sigma'=P_n$

$$\boxed{C=\varepsilon_r C_0} \quad \boxed{E=E_0/\varepsilon_r}$$ （均匀介质）

注意

有介质时先求 $\vec{D}\rightarrow\vec{E}\rightarrow U$

例1 把一块相对电容率 $\varepsilon_r = 3$ 的电介质,放在极板间相距 $d = 1\mathrm{mm}$ 的平行平板电容器的两极板之间.放入之前,两极板的电势差是 $1000V$．试求两极板间电介质内的电场强度 E ,电极化强度 P ,极板和电介质的电荷面密度,电介质内的电位移 D .

解 $$E_0 = \frac{U}{d} = \frac{1000}{10^{-3}}\mathrm{V\cdot m^{-1}} = 10^6\mathrm{V\cdot m^{-1}} = 10^3\mathrm{kV\cdot m^{-1}}$$

$$E = E_0/\varepsilon_r = 3.33\times10^2\mathrm{kV\cdot m^{-1}}$$

$$P = (\varepsilon_r - 1)\varepsilon_0 E = 5.89\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$\sigma_0 = \varepsilon_0 E_0 = 8.85\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$\sigma' = P = 5.89\times10^{-6}\mathrm{C\cdot m^{-2}}$$

$$D = \varepsilon_0\varepsilon_r E = \varepsilon_0 E_0 = \sigma_0 = 8.85\times10^{-6}\mathrm{C\cdot m^{-2}}$$

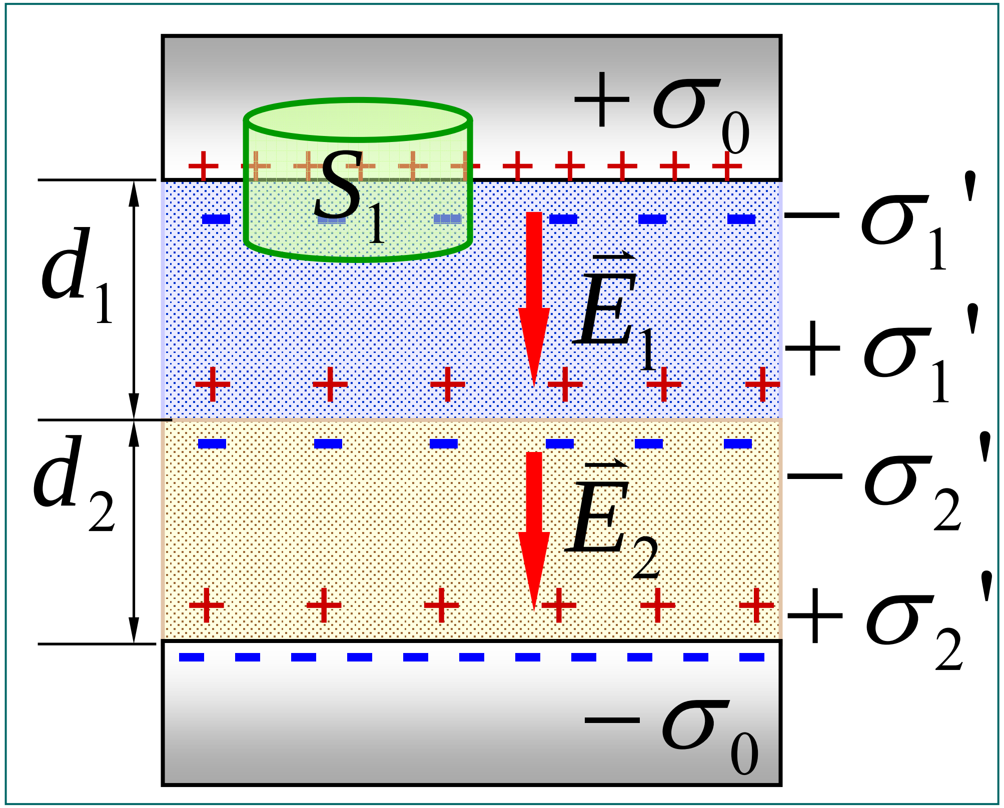
例2 一平行平板电容器充满两层厚度各为 d_1 和 d_2 的电介质，它们的相对电容率分别为 ε_{r1} 和 ε_{r2}，极板面积为 S． 求（1）电容器的电容；（2）当极板上的自由电荷面密度的值为 σ_0 时，两介质分界面上的极化电荷面密度.

解（1）$\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \sigma_0 S_1$

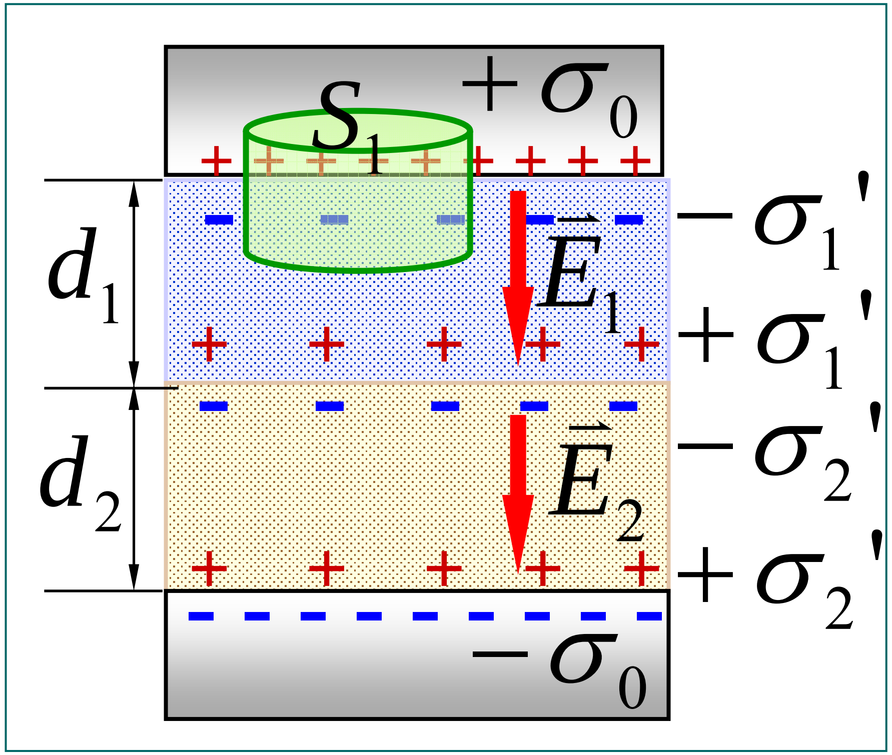
$$D = \sigma_0$$

$$E_1 = \frac{D}{\varepsilon_0\varepsilon_{r1}} = \frac{\sigma_0}{\varepsilon_0\varepsilon_{r1}}$$

$$E_2 = \frac{D}{\varepsilon_0\varepsilon_{r2}} = \frac{\sigma_0}{\varepsilon_0\varepsilon_{r2}}$$



$$\begin{cases} E_1 = \dfrac{D}{\varepsilon_0 \varepsilon_{r1}} = \dfrac{\sigma_0}{\varepsilon_0 \varepsilon_{r1}} \\ E_2 = \dfrac{D}{\varepsilon_0 \varepsilon_{r2}} = \dfrac{\sigma_0}{\varepsilon_0 \varepsilon_{r2}} \end{cases}$$



$$U = \int_l \vec{E} \cdot \mathrm{d}\vec{l} = E_1 d_1 + E_2 d_2$$

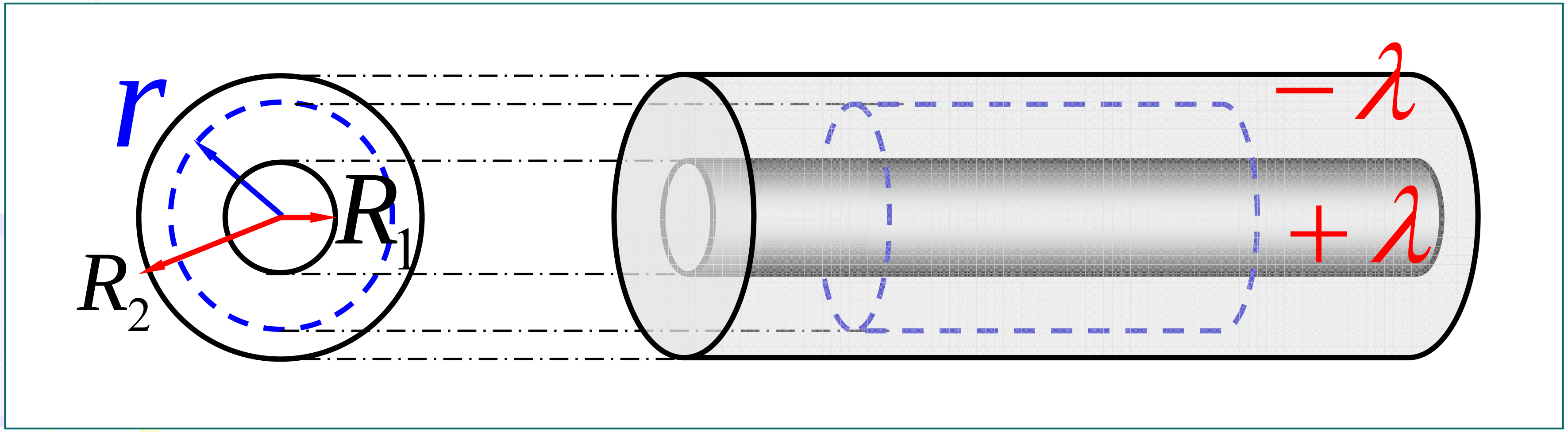
$$= \frac{Q}{\varepsilon_0 S}\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}}\right)$$

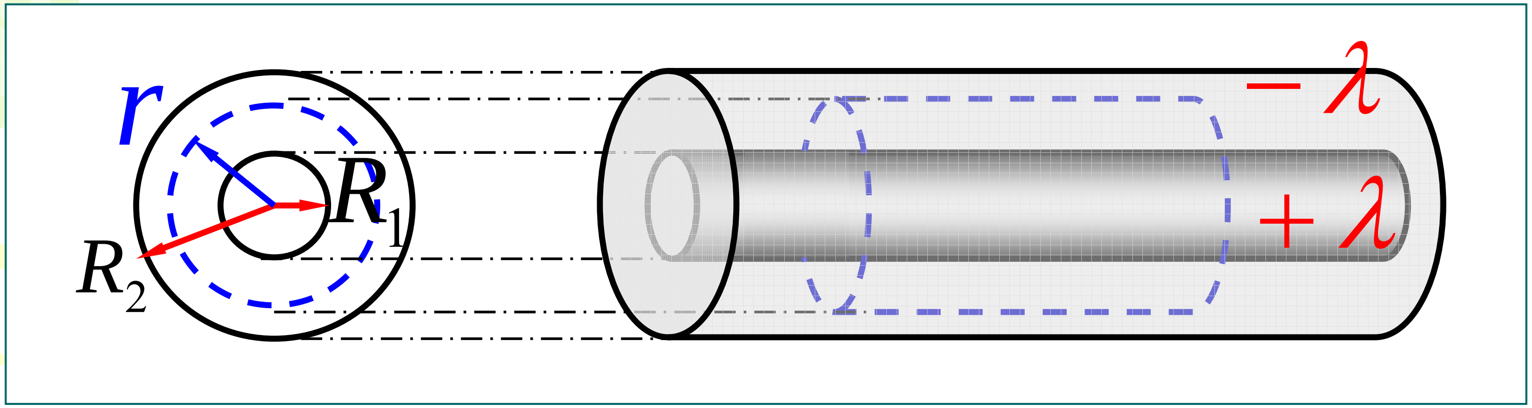
$$C = \frac{Q_0}{U} = \frac{\varepsilon_0 \varepsilon_{r1} \varepsilon_{r2} S}{\varepsilon_{r1} d_2 + \varepsilon_{r2} d_1}$$

（2） $\sigma_1' = \dfrac{\varepsilon_{r1} - 1}{\varepsilon_{r1}} \sigma_0$

$$\sigma_2' = \frac{\varepsilon_{r2} - 1}{\varepsilon_{r2}} \sigma_0$$

例3 常用的圆柱形电容器，是由半径为 R_1 的长直圆柱导体和同轴的半径为 R_2 的薄导体圆筒组成，并在直导体与导体圆筒之间充以相对电容率为 ε_r 的电介质.设直导体和圆筒单位长度上的电荷分别为 $+\lambda$ 和 $-\lambda$ ． 求（1）电介质中的电场强度、电位移和极化强度；（2）电介质内、外表面的极化电荷面密度；（3）此圆柱形电容器的电容.





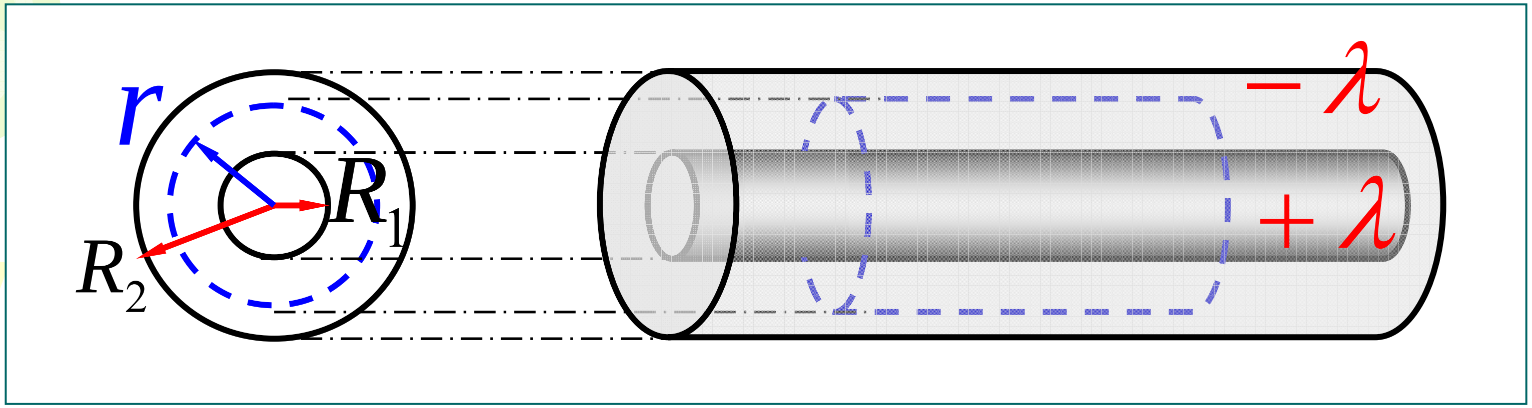
解（1）

$$\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \lambda l$$

$$D 2\pi r l = \lambda l \qquad D = \frac{\lambda}{2\pi r}$$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_r r} \qquad (R_1 < r < R_2)$$

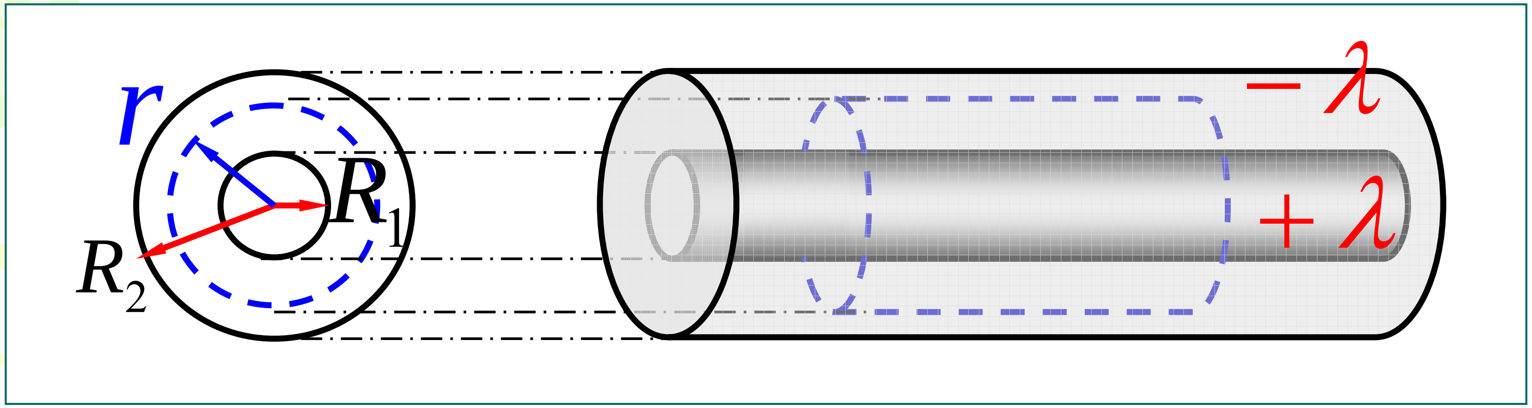
$$P = (\varepsilon_r - 1)\varepsilon_0 E = \frac{\varepsilon_r - 1}{2\pi \varepsilon_r r}\lambda$$



（2）由上题可知

$$E=\frac{D}{\varepsilon_0\varepsilon_r}=\frac{\lambda}{2\pi\ \varepsilon_0\varepsilon_r r}\qquad \begin{cases} E_1=\dfrac{\lambda}{2\pi\ \varepsilon_0\varepsilon_r R_1} & (r=R_1) \\[2ex] E_2=\dfrac{\lambda}{2\pi\ \varepsilon_0\varepsilon_r R_2} & (r=R_2) \end{cases}$$

$$\begin{cases} \sigma_1{}'=(\varepsilon_r-1)\varepsilon_0 E_1=(\varepsilon_r-1)\dfrac{\lambda}{2\pi\ \varepsilon_r R_1} \\[2ex] \sigma_2{}'=(\varepsilon_r-1)\varepsilon_0 E_2=(\varepsilon_r-1)\dfrac{\lambda}{2\pi\ \varepsilon_r R_2} \end{cases}$$



（3）由（1）可知 $E=\dfrac{\lambda}{2\pi\,\varepsilon_0\varepsilon_r r}\quad (R_1<r<R_2)$

$$U=\int \vec{E}\cdot \mathrm{d}\vec{r}=\int_{R_1}^{R_2}\frac{\lambda\,\mathrm{d}r}{2\pi\,\varepsilon_0\varepsilon_r r}=\frac{\lambda}{2\pi\,\varepsilon_0\varepsilon_r}\ln\frac{R_2}{R_1}$$

$$C=\frac{Q}{U}=2\pi\,\varepsilon_0\varepsilon_r l\Big/\ln\frac{R_2}{R_1}=\varepsilon_r\boxed{C_0}$$

真空圆柱形电容器电容

单位长度电容 $\dfrac{C}{l}=2\pi\,\varepsilon_0\varepsilon_r\Big/\ln\dfrac{R_2}{R_1}$