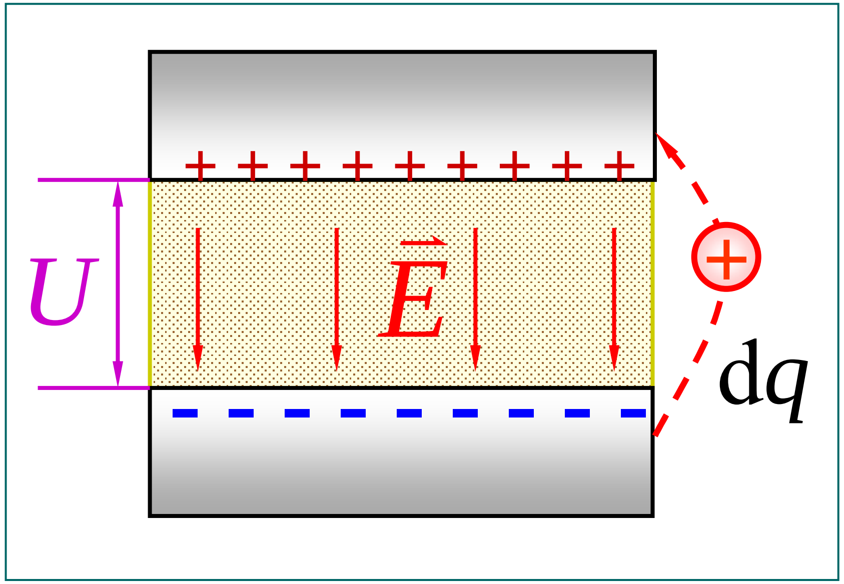
一　电容器的电能

$$\mathrm{d}W = U\mathrm{d}q = \frac{q}{C}\mathrm{d}q$$

$$W = \frac{1}{C}\int_0^Q q\mathrm{d}q = \frac{Q^2}{2C}$$

$$C = \frac{Q}{U}$$

$$W = \frac{1}{2}QU = \frac{1}{2}CU^2$$



电容器贮存的电能　$W_\mathrm{e} = \frac{Q^2}{2C} = \frac{1}{2}QU = \frac{1}{2}CU^2$

二 静电场的能量 能量密度

$$W_e=\frac{1}{2}CU^2=\frac{1}{2}\frac{\varepsilon S}{d}(Ed)^2=\frac{1}{2}\varepsilon E^2 Sd$$

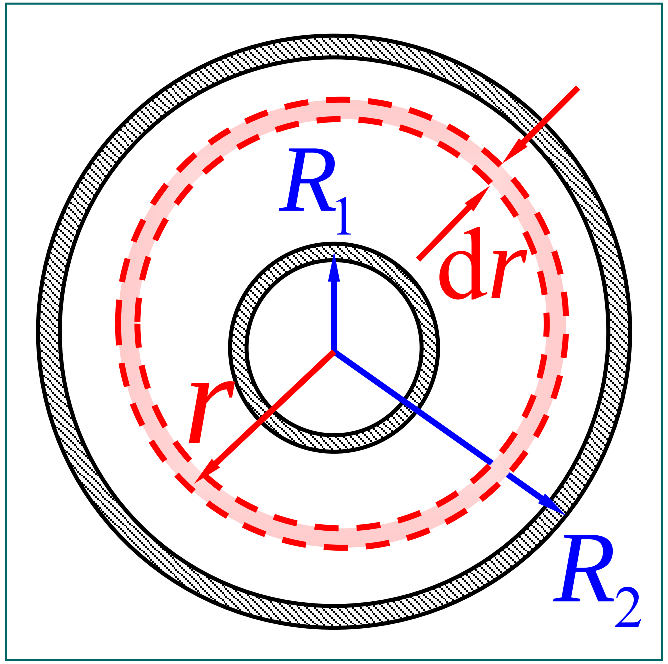
电场能量密度 $w_e=\frac{1}{2}\varepsilon E^2=\frac{1}{2}ED$

物理意义 电场是一种物质，它具有能量.

电场空间所存储的能量

$$W_e=\int_V w_e \mathrm{d}V=\int_V \frac{1}{2}\varepsilon E^2 \mathrm{d}V$$

例1　如图所示，球形电容器的内、外半径分别为 R_1 和 R_2，所带电荷为 $\pm Q$．若在两球壳间充以电容率为 ε 的电介质，问此电容器贮存的电场能量为多少？



解　$$\vec{E}=\frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\vec{e}_r$$

$$w_e=\frac{1}{2}\varepsilon E^2=\frac{Q^2}{32\pi^2\varepsilon r^4}$$

$$\mathrm{d}W_e=w_e\mathrm{d}V=\frac{Q^2}{8\pi\varepsilon r^2}\mathrm{d}r$$

$$W_e=\int\mathrm{d}W_e=\frac{Q^2}{8\pi\varepsilon}\int_{R_1}^{R_2}\frac{\mathrm{d}r}{r^2}=\frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$W_e=\frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{2}\frac{Q^2}{4\pi\varepsilon\dfrac{R_2R_1}{R_2-R_1}}$$

讨论

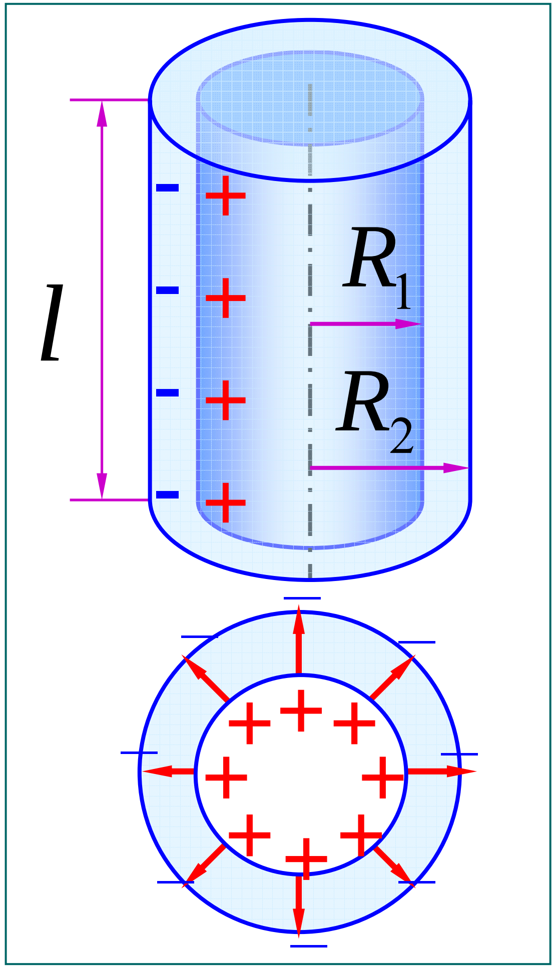
（1） $W_e=\dfrac{Q^2}{2C}$ $C=4\pi\varepsilon\dfrac{R_2R_1}{R_2-R_1}$

（球形电容器电容）

（2） $R_2\rightarrow\infty$ $W_e=\dfrac{Q^2}{8\pi\varepsilon R_1}$

（孤立导体球贮存的能量）

例2 如图圆柱形电容器，中间是空气，空气的击穿场强是 $E_b = 3\times10^6\,\mathrm{V\cdot m^{-1}}$，电容器外半径 $R_2 = 10^{-2}\,\mathrm{m}$. 在空气不被击穿的情况下，内半径 $R_1 = ?$ 可使电容器存储能量最多.（空气 $\varepsilon_r \approx 1$ ）



解
$$E = \frac{\lambda}{2\pi\,\varepsilon_0 r} \quad (R_1 < r < R_2)$$

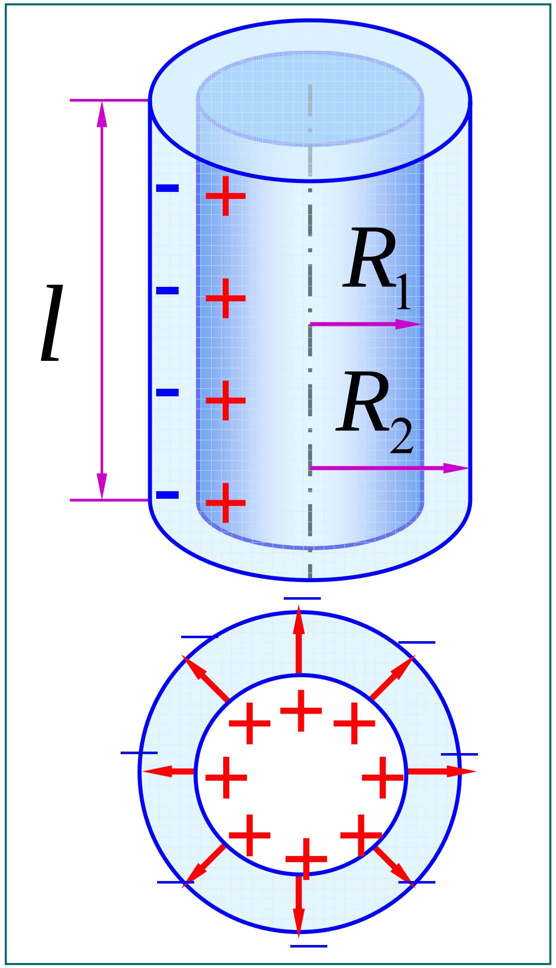
$$E_b = \frac{\lambda_{max}}{2\pi\,\varepsilon_0 R_1}$$

$$U = \frac{\lambda}{2\pi\,\varepsilon_0}\int_{R_1}^{R_2}\frac{\mathrm{d}r}{r} = \frac{\lambda}{2\pi\,\varepsilon_0}\ln\frac{R_2}{R_1}$$

单位长度的电场能量

$$W_e = \frac{1}{2}\lambda U = \frac{\lambda^2}{4\pi\,\varepsilon_0}\ln\frac{R_2}{R_1}$$

$$W_e = \frac{\lambda^2}{4\pi\varepsilon_0}\ln\frac{R_2}{R_1} \qquad E_b = \frac{\lambda_{max}}{2\pi\varepsilon_0 R_1}$$



$$\lambda = \lambda_{max} = 2\pi\varepsilon_0 E_b R_1$$

$$W_e = \pi\varepsilon_0 E_b^2 R_1^2 \ln\frac{R_2}{R_1}$$

$$\frac{dW_e}{dR_1} = \pi\varepsilon_0 E_b^2 R_1 (2\ln\frac{R_2}{R_1} - 1) = 0$$

$$R_1 = \frac{R_2}{\sqrt{e}} = \frac{10^{-2}}{\sqrt{e}}\,\mathrm{m} \approx 6.07\times10^{-3}\,\mathrm{m}$$

$$U_{max} = E_b R_1 \ln\frac{R_2}{R_1} = \frac{E_b R_2}{2\sqrt{e}} = 9.10\times10^3\,\mathrm{V}$$