

一 单摆 $\theta < 5^\circ$ 时, $\sin\theta \approx \theta$

$$M = -mgl \sin\theta \approx -mgl\theta$$

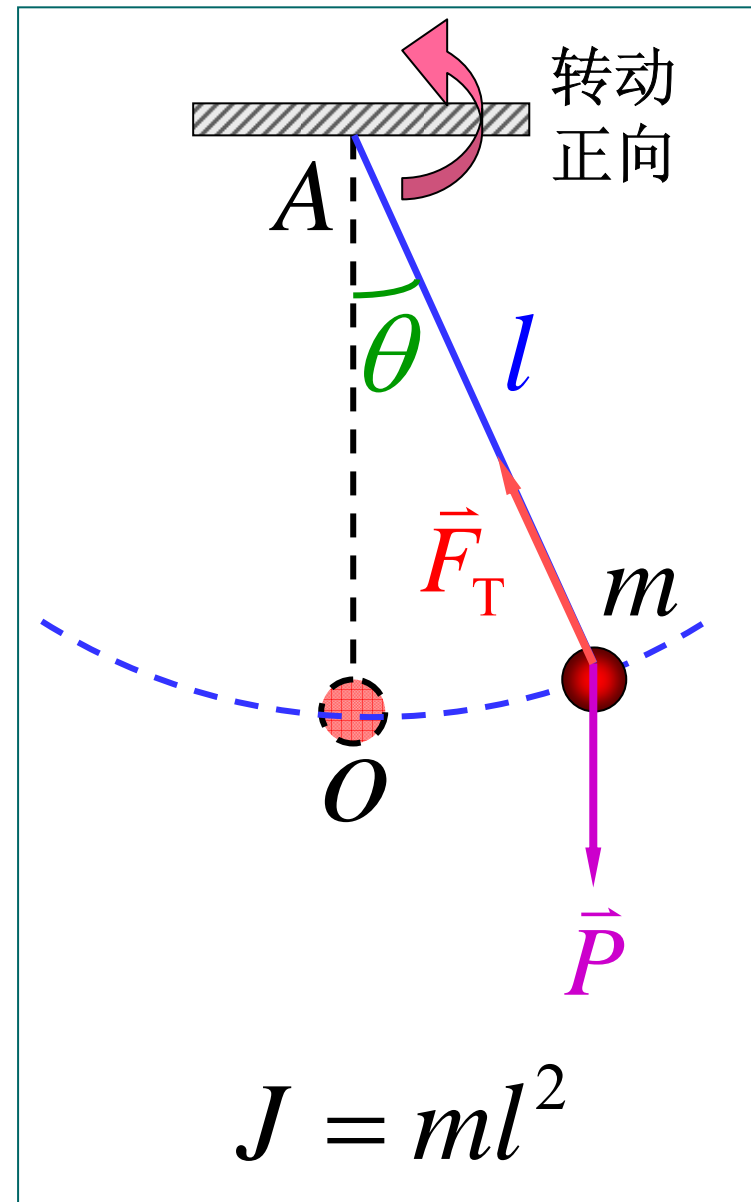
$$-mgl\theta = J \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \quad \text{令 } \omega^2 = \frac{g}{l}$$

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\theta = \theta_m \cos(\omega t + \varphi)$$

$$T = 2\pi \sqrt{l/g}$$



二 复摆 ($\theta < 5^\circ$)

$$M \approx -mgl \theta$$

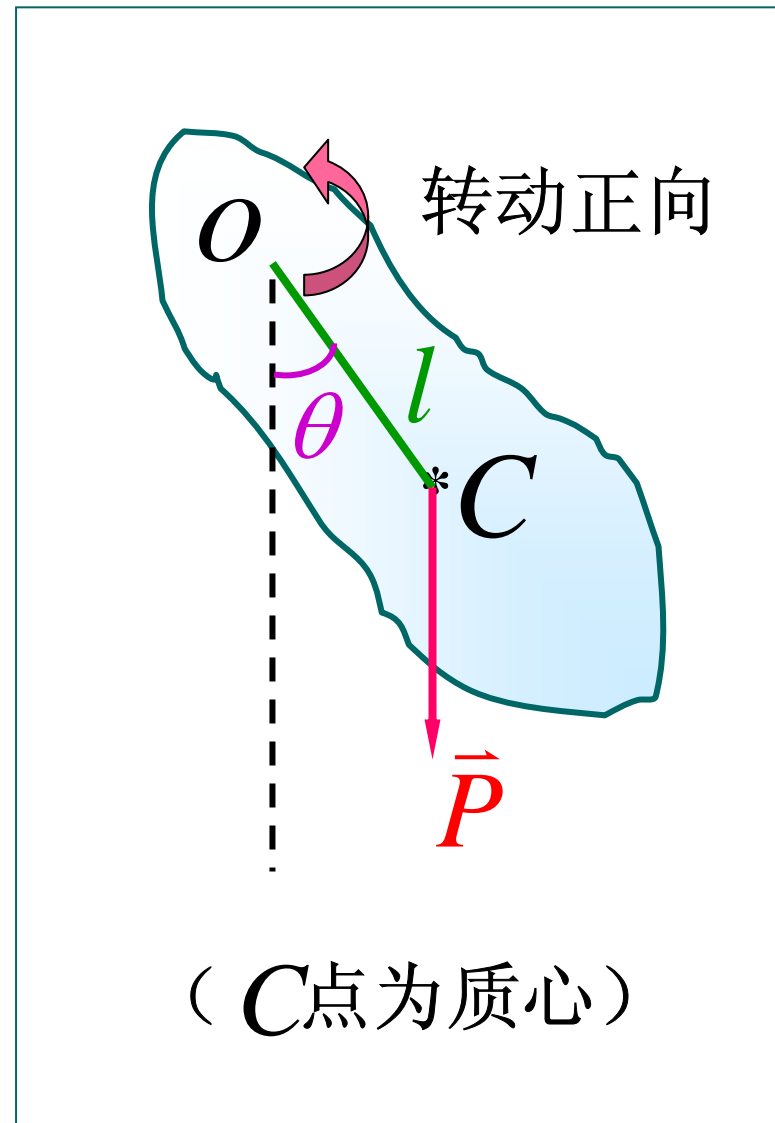
$$-mgl \theta = J \frac{d^2 \theta}{dt^2}$$

$$\text{令 } \omega^2 = \frac{mgl}{J}$$

$$\frac{d^2 \theta}{dt^2} = -\omega^2 \theta$$

$$\theta = \theta_m \cos(\omega t + \varphi)$$

$$T = 2\pi \sqrt{\frac{J}{mgl}}$$



三 简谐运动的描述和特征

1) 物体受线性回复力作用 $F = -kx$ 平衡位置 $x = 0$

2) 简谐运动的动力学描述

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

3) 简谐运动的运动学描述

$$x = A \cos(\omega t + \varphi)$$

$$v = -A \omega \sin(\omega t + \varphi)$$

4) 加速度与位移成正比而方向相反 $a = -\omega^2 x$

弹簧振子 $\omega = \sqrt{k/m}$

单摆 $\omega = \sqrt{g/l}$

复摆 $\omega = \sqrt{mgl/J}$