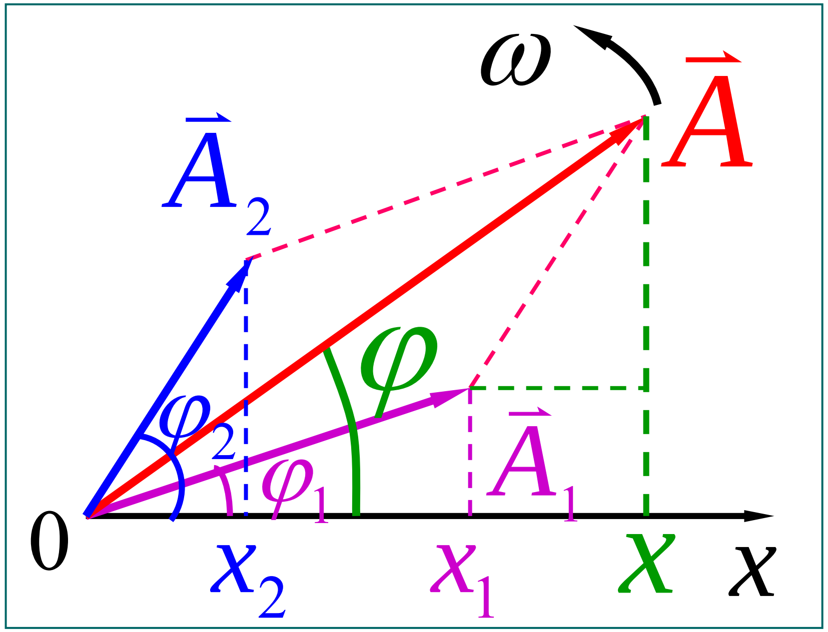
一　两个同方向同频率简谐运动的合成

$$\begin{cases} x_1 = A_1\cos(\omega t + \varphi_1) \\ x_2 = A_2\cos(\omega t + \varphi_2) \end{cases}$$

$$x = x_1 + x_2$$

$$\boxed{x = A\cos(\omega t + \varphi)}$$



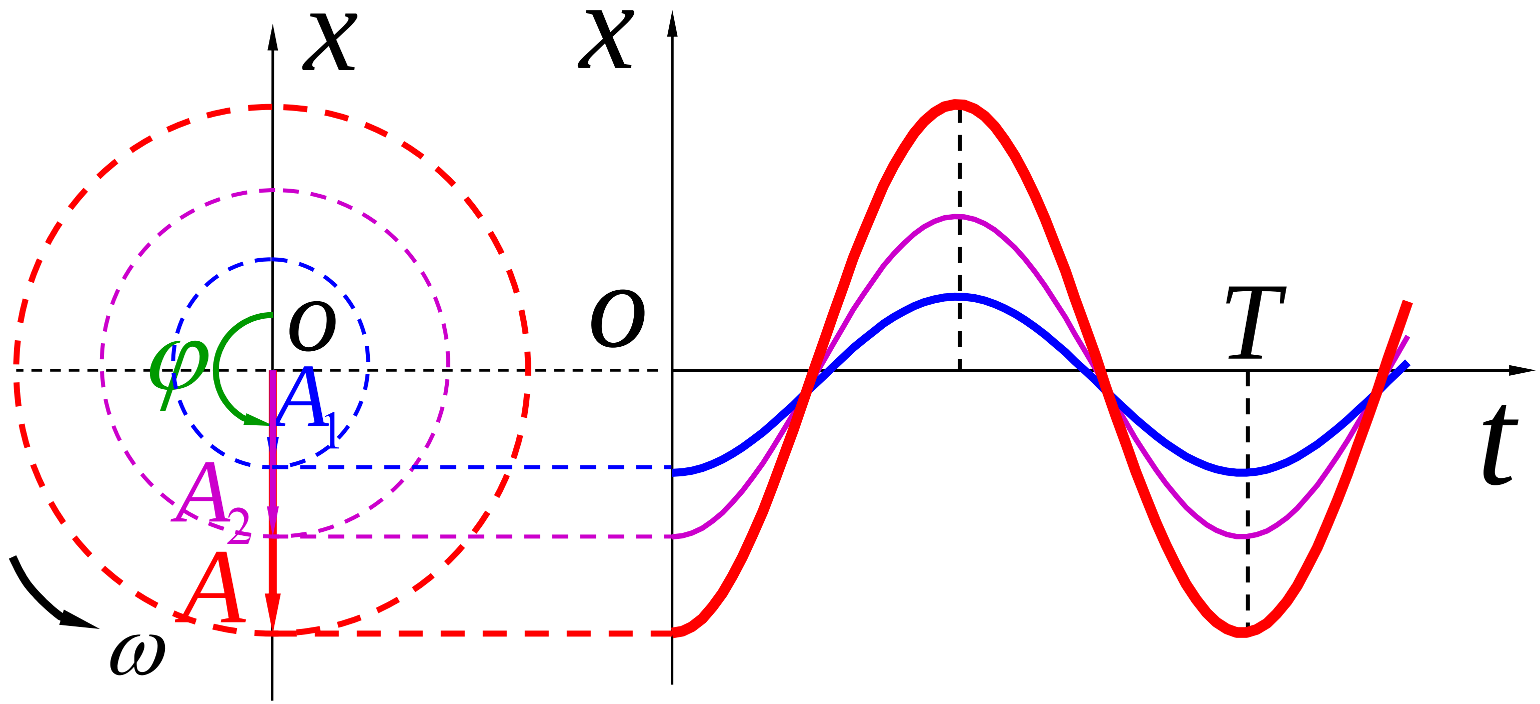
$$\begin{cases} A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)} \\ \tan\varphi = \dfrac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \end{cases}$$

两个同方向同频率简谐运动合成后仍为简谐运动

讨论

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

1）相位差 $\Delta\varphi=\varphi_2-\varphi_1=2k\pi\quad(k=0,\pm1,\pm2,\cdots)$



$$\begin{cases}A=A_1+A_2\\ \varphi=\varphi_2=\varphi_1+2k\pi\end{cases}$$

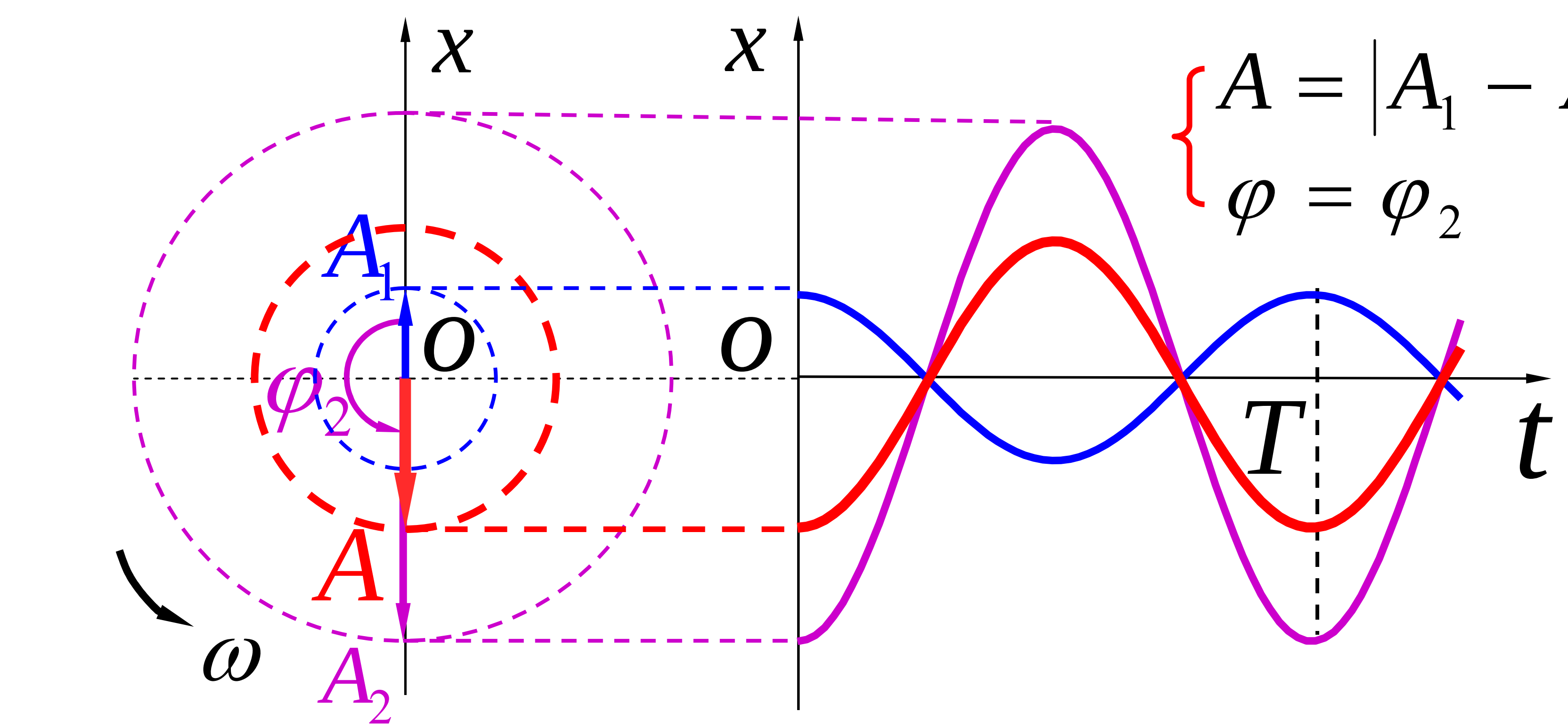
$$\boxed{x=(A_1+A_2)\cos(\omega t+\varphi)}$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

2）相位差 $\Delta\varphi=\varphi_2-\varphi_1=(2k+1)\pi \quad (k=0,\pm1,\cdots)$

$$\begin{cases} x_1 = A_1\cos\omega t \\ x_2 = A_2\cos(\omega t+\pi) \end{cases}$$

$$\boxed{x=(A_2-A_1)\cos(\omega t+\pi)}$$



$$\begin{cases} A=\left|A_1-A_2\right| \\ \varphi=\varphi_2 \end{cases}$$

1）相位差 $\varphi_2 - \varphi_1 = 2k\pi \qquad (k = 0, \pm 1, \cdots)$

$$A = A_1 + A_2$$

相互加强

2）相位差 $\varphi_2 - \varphi_1 = (2k+1)\pi \quad (k = 0, \pm 1, \cdots)$

$$A = \left| A_1 - A_2 \right|$$

相互削弱

3）一般情况

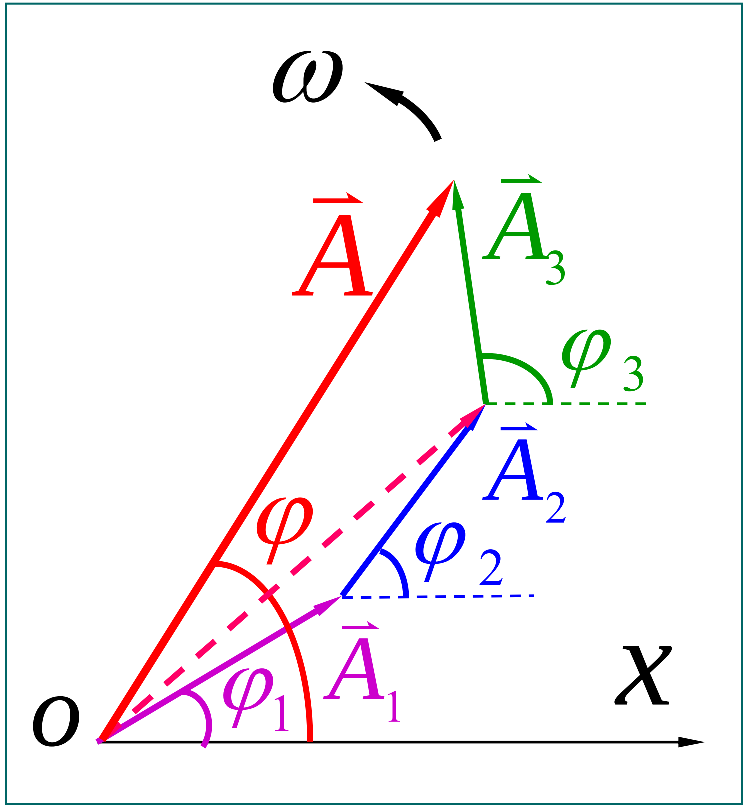
$$A_1 + A_2 > A > \left| A_1 - A_2 \right|$$

二 多个同方向同频率简谐运动的合成

$$\begin{cases} x_1 = A_1\cos(\omega t + \varphi_1) \\ x_2 = A_2\cos(\omega t + \varphi_2) \\ \cdots\cdots \\ x_n = A_n\cos(\omega t + \varphi_n) \end{cases}$$

$$x = x_1 + x_2 + \cdots + x_n$$

$$\boxed{x = A\cos(\omega t + \varphi)}$$



多个同方向同频率简谐运动合成仍为简谐运动

$$\begin{cases} x_1 = A_0 \cos \omega t \\ x_2 = A_0 \cos(\omega t + \Delta\varphi) \\ x_3 = A_0 \cos(\omega t + 2\Delta\varphi) \\ \quad \cdots\cdots \\ x_N = A_0 \cos[\omega t + (N-1)\Delta\varphi] \end{cases}$$

讨论

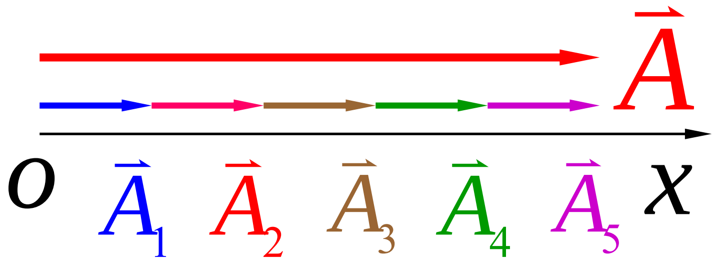
1） $\Delta\varphi = 2k\pi$

$(k = 0, \pm 1, \pm 2, \cdots)$

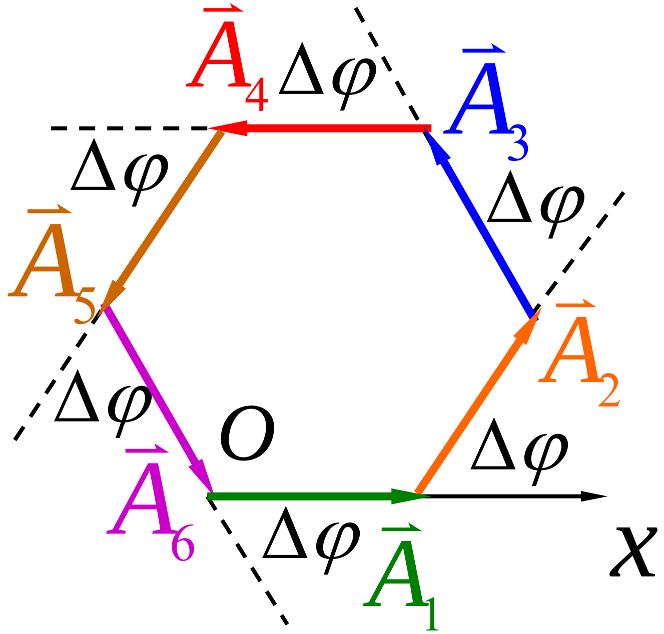
2） $N\Delta\varphi = 2k'\pi$

$(k' \neq kN, k' = \pm 1, \pm 2, \cdots)$

N个矢量依次相接构成一个闭合的多边形 .

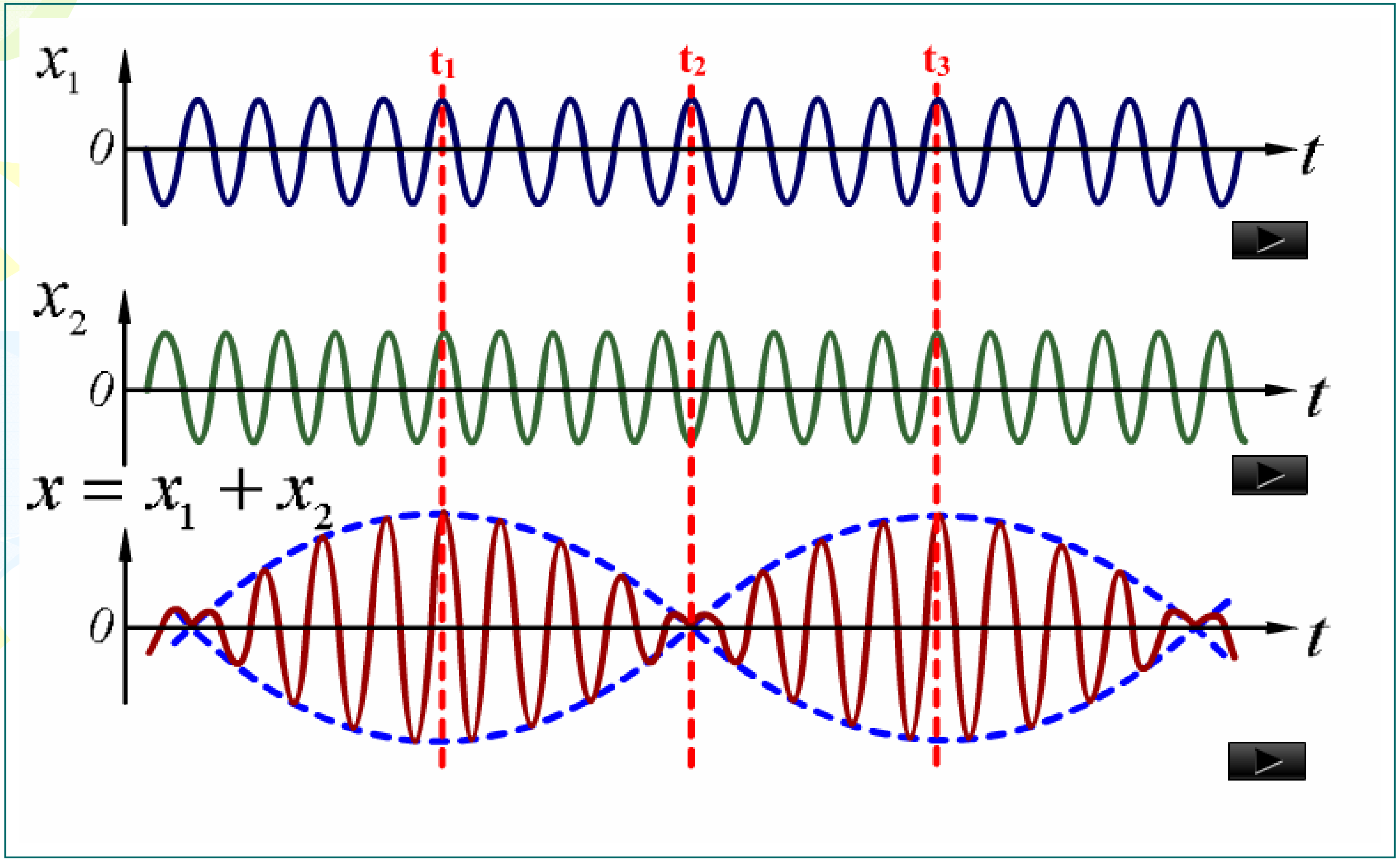


$$A = \sum_i A_i = NA_0$$



$$A = 0$$

三 两个同方向不同频率简谐运动的合成



频率较大而频率之差很小的两个同方向简谐运动的合成，其合振动的振幅时而加强时而减弱的现象叫拍.

$$\begin{cases} x_1 = A_1 \cos \omega_1 t = A_1 \cos 2\pi\ \nu_1 t \\ x_2 = A_2 \cos \omega_2 t = A_2 \cos 2\pi\ \nu_2 t \end{cases}$$

$$\boxed{x = x_1 + x_2}$$

讨论 $A_1 = A_2,\ \left|\nu_2 - \nu_1\right| << \nu_1 + \nu_2$ 的情况

◆ 方法一

$$x = x_1 + x_2 = A_1 \cos 2\pi\ \nu_1 t + A_2 \cos 2\pi\ \nu_2 t$$

$$x = \underbrace{\left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right)}_{\text{振幅部分}} \cos 2\pi \underbrace{\frac{\nu_2 + \nu_1}{2}}_{\text{合振动频率}} t$$

$$x = \underbrace{\left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right)}_{\text{振幅部分}} \cos 2\pi \underbrace{\frac{\nu_2 + \nu_1}{2}}_{\text{合振动频率}} t$$

振动频率 $\nu = (\nu_1 + \nu_2)/2$

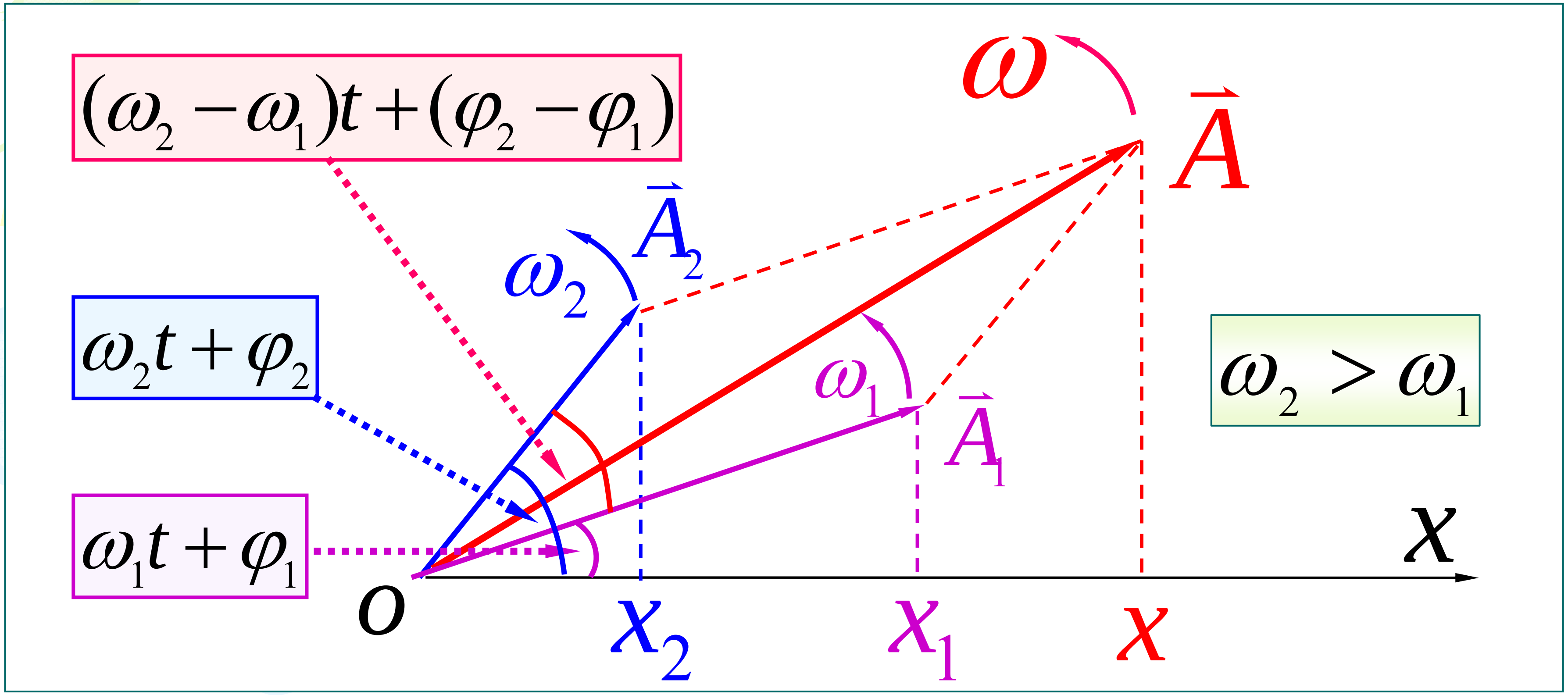
振幅 $A = \left| 2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t \right|$

$$\begin{cases} A_{max} = 2A_1 \\ A_{min} = 0 \end{cases}$$

$$2\pi \frac{\nu_2 - \nu_1}{2} T = \pi \qquad T = \frac{1}{\nu_2 - \nu_1}$$

$$\boxed{\nu = \nu_2 - \nu_1}$$ ⇐ 拍频（振幅变化的频率）

◆ 方法二：旋转矢量合成法



$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\varphi}\qquad \varphi_1=\varphi_2=0$$

$$\Delta\varphi=(\omega_2-\omega_1)t+(\varphi_2-\varphi_1)\qquad \Delta\varphi=2\pi(\nu_2-\nu_1)t$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\varphi}$$

$$\Delta\varphi=(\omega_2-\omega_1)t$$

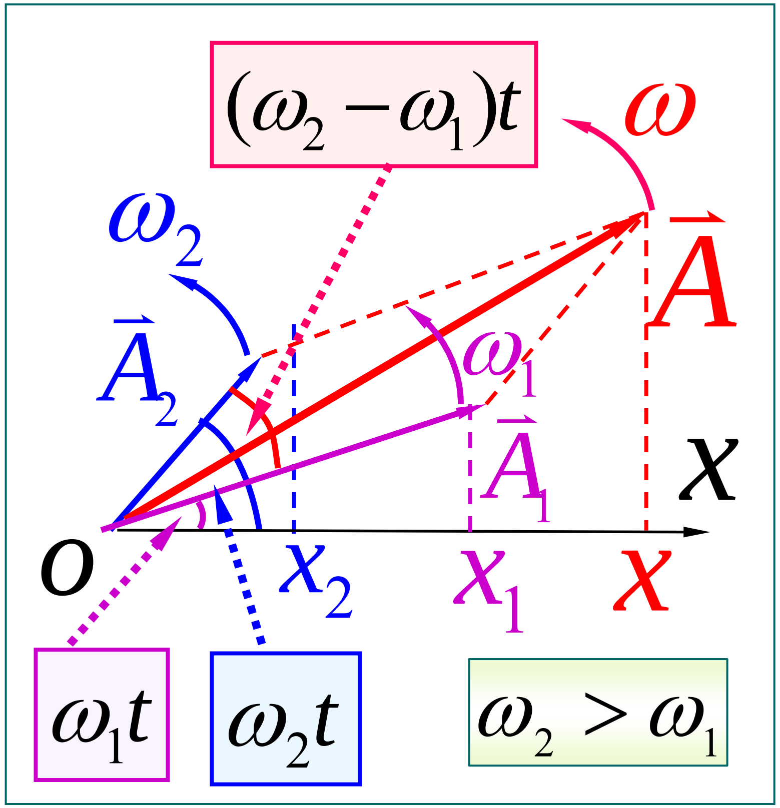
振幅 $A=A_1\sqrt{2(1+\cos\Delta\varphi)}$

$$=\left|2A_1\cos\left(\frac{\omega_2-\omega_1}{2}t\right)\right|$$

拍频 $\Rightarrow$ $\boxed{\nu=\nu_2-\nu_1}$

（拍在声学和无线电技术中的应用）

振动圆频率　$\cos\omega t=\dfrac{x_1+x_2}{A}$　　$\omega=\dfrac{\omega_1+\omega_2}{2}$



四 两个相互垂直的同频率简谐运动的合成

$$\begin{cases} x = A_1 \cos(\omega t + \varphi_1) \\ y = A_2 \cos(\omega t + \varphi_2) \end{cases}$$

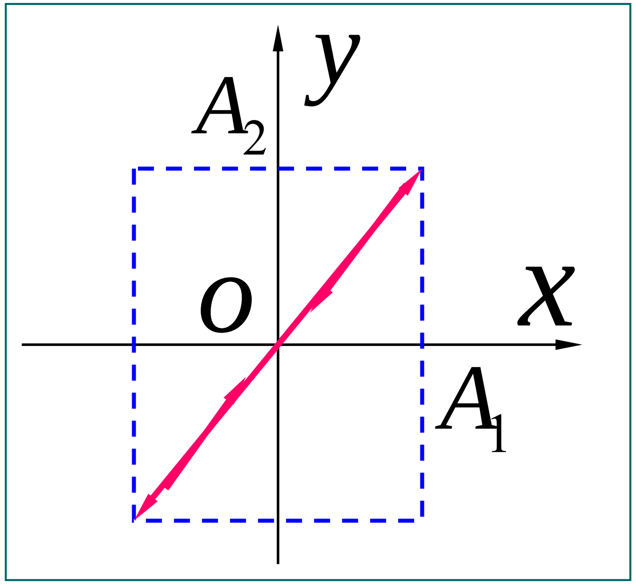
质点运动轨迹 （椭圆方程）

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$$

讨论

1） $\varphi_2 - \varphi_1 = 0$ 或 2π

$$y = \frac{A_2}{A_1} x$$



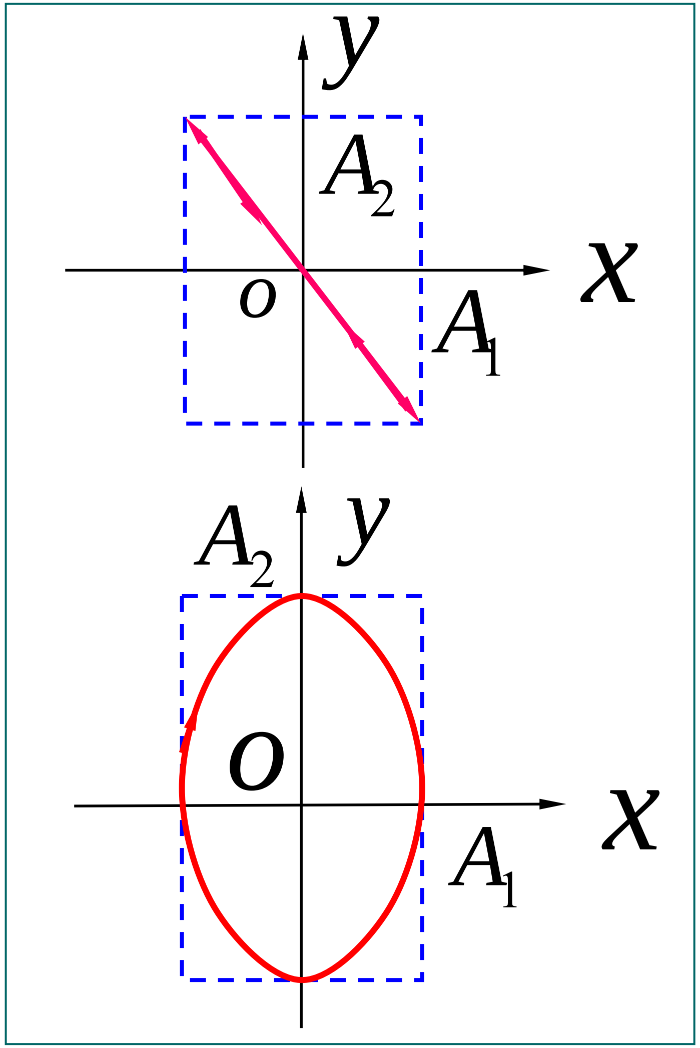
$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

2） $\varphi_2-\varphi_1=\pi$ $y=-\dfrac{A_2}{A_1}x$

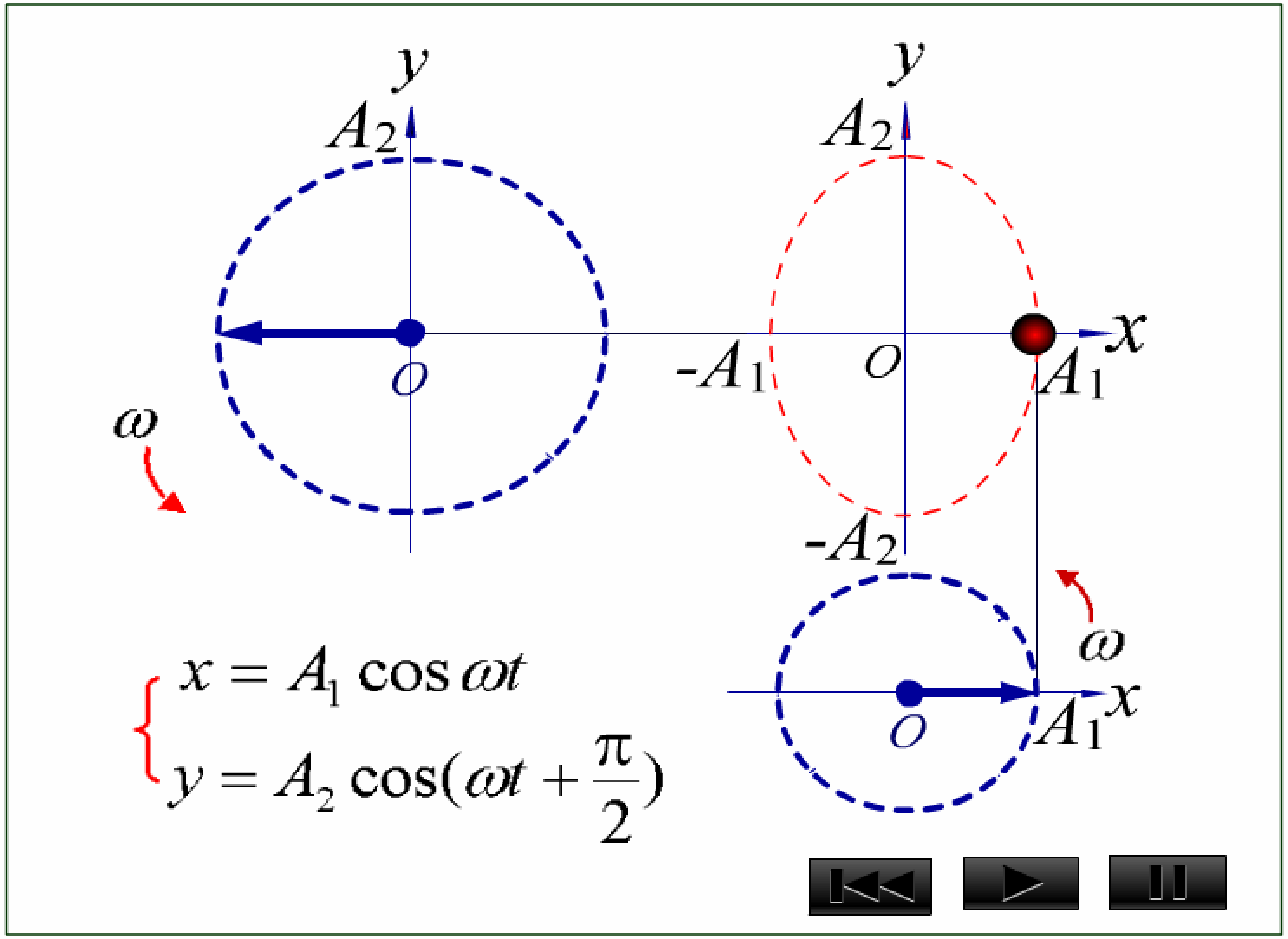
3） $\varphi_2-\varphi_1=\pm\pi/2$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

$$\begin{cases} x=A_1\cos\omega t \\ y=A_2\cos(\omega t+\dfrac{\pi}{2}) \end{cases}$$

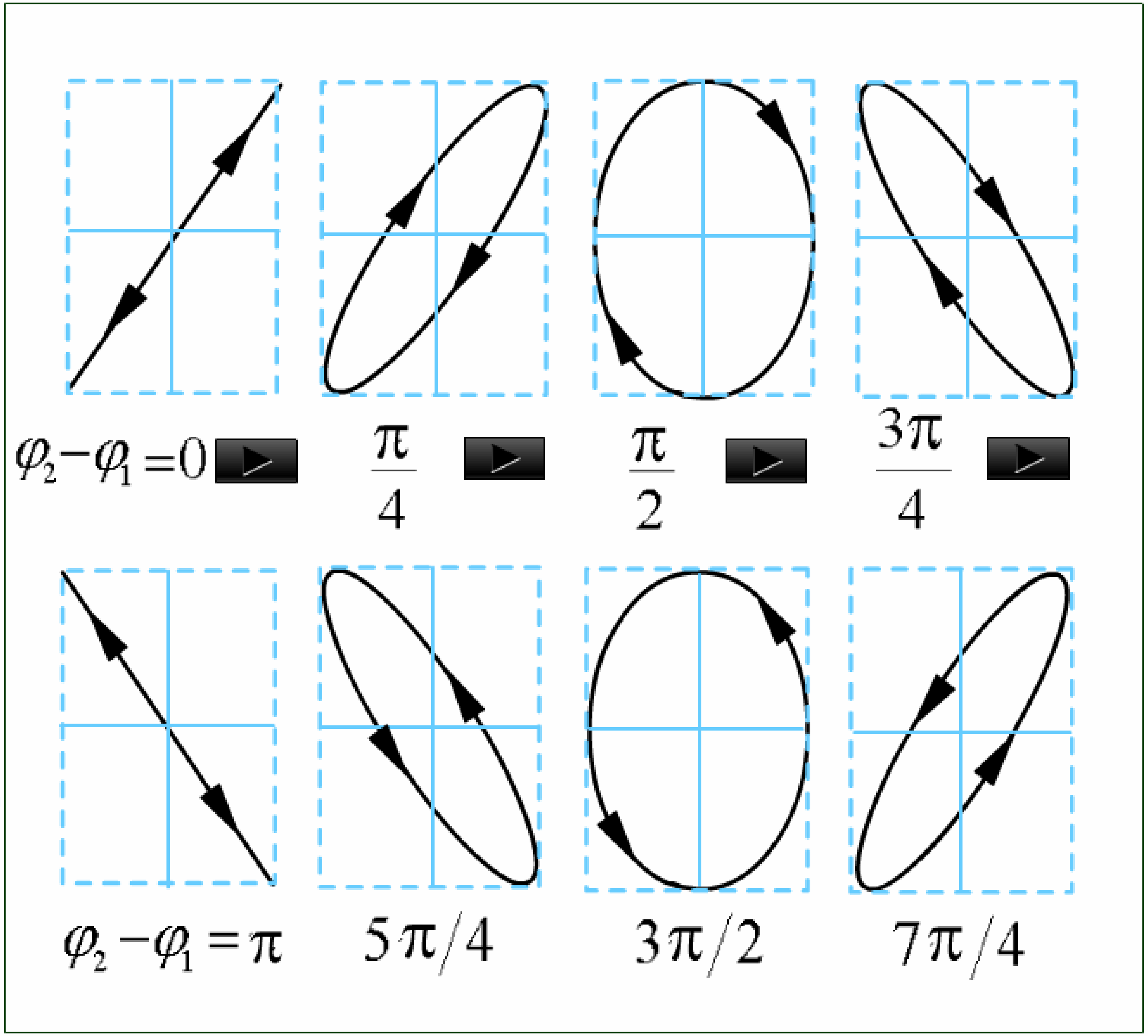


用旋转矢量描绘振动合成图



$$\begin{cases} x = A_1 \cos \omega t \\ y = A_2 \cos(\omega t + \dfrac{\pi}{2}) \end{cases}$$

两相互垂直同频率不同相位差简谐运动的合成图



五 两相互垂直不同频率的简谐运动的合成

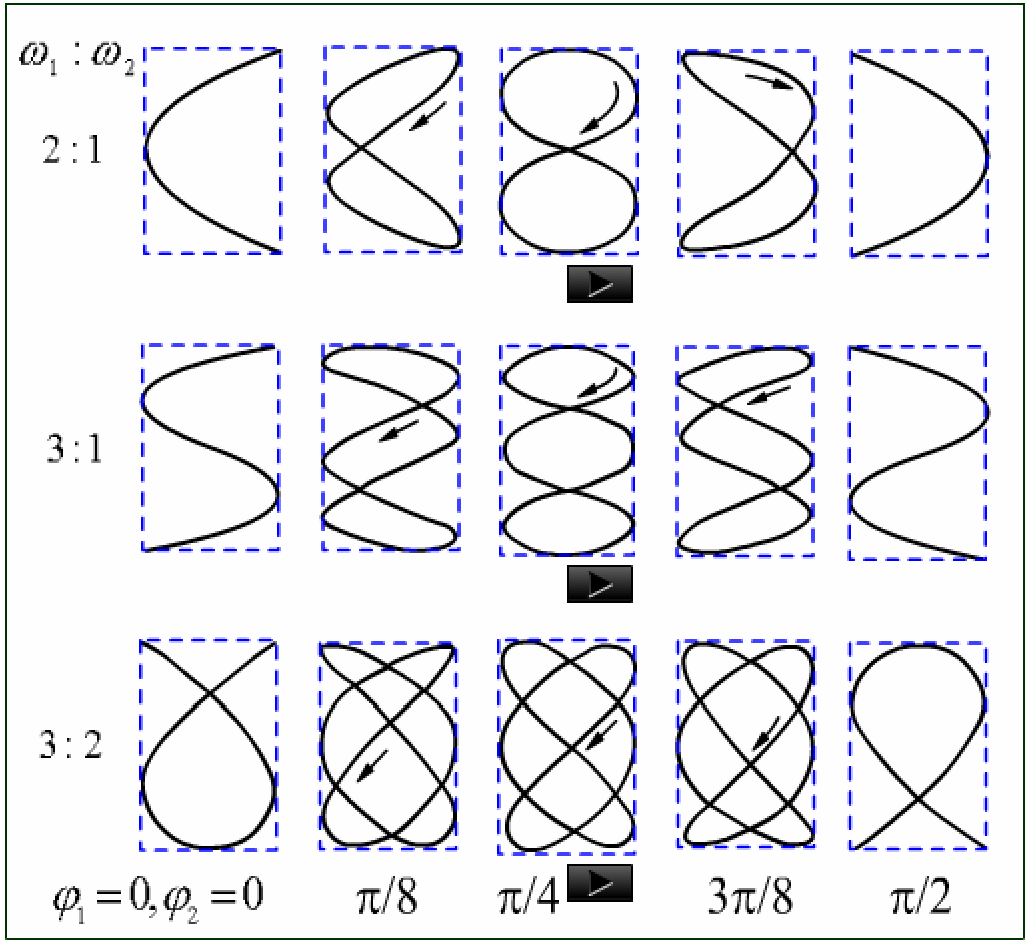
$$\begin{cases} x = A_1\cos(\omega_1 t + \varphi_1) \\ y = A_2\cos(\omega_2 t + \varphi_2) \end{cases}$$

李 萨 如 图

$$\varphi_1 = 0$$

$$\varphi_2 = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$$

$$\frac{\omega_1}{\omega_2} = \frac{m}{n}$$



测量振动频率和相位的方法